University Of Anbar
College of Engineering
Mechanical Engineering Department
First class

Lecturers

physics - I - (Mechanics and Thermodynamics)

By

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Physics I - Mechanics and Thermodynamics

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Lecture

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Chap. 1
Physics and Measurement

classical physics

Includes the theories, concept laws and experiments in classical mechanics, thermodynamics, optics and electromagnetism developed before 1900.

Modern physics Began near the end of the 19th century. The two most important developments in this modern era were the theories of relativity and quantum mechanics. Ex:- Einstein's theory of relativity.

Standards of length, Mass and Time

In mechanics, the three basic quantities are length, mass and time. In 1960, an international committee established a set of standards for the fundamental quantities of science. It is called the SI (systeme International) and it is units of length, mass and time are the (meter, kilogram, and second) respectively. other SI standards established by the committee are those for temperature (the kelvin), electric current (the ampere), luminous intensity (the candela), and the amount of substance (the mole).

Length

As recently as 1960, the length of the meter was defined as (the distance between two lines on a specific platinum-iridium bar stored under controlled conditions in France). In 1960s and 1970s, the meter was defined as (1 650 763.73 wave lengths of orange-red light emitted from a krypton-86 lamp). In 1983, the meter (m) was defined as (the distance traveled by light in vacuum during a time of 1/299 792 458 second). This last definition established that the speed of light in vacuum is precisely (299 792 458 meters per second.

Examples

Mean radius of the Earth = $6.37*10^6$ m

Mean distance from the Earth to the Moon = $3.84*10^8$ m

one-light year = $9.46*10^{15}$ m

size of smallest dust particles $\approx 10^{-4}$ m

Diameter of a hydrogen atom $\simeq 10^{-10}$ m

Mass

The SI unit of mass, the kilogram (kg); is defined as (the mass of a specific platinum - iridium alloy cylinder kept at the international Bureau of weights and Measures at sevres, France). This mass standard was established in 1887.

Examples

Sun mass = $1.99*10^{30}$ kg, Earth mass = $5.98*10^{24}$ kg

Bacterium mass $\simeq 1*10^{-15}$ kg

Time

Before, 1960, the standard of time was defined in terms of the (mean solar day) for the year 1900. (A solar day) is (the time interval between successive appearances of the Sun at the highest point it reaches in the sky each day).

The second was defined as (1/60)(1/60)(1/24) of a mean solar day. The rotation of the Earth is now known to vary slightly with time, however, and therefore this motion is not a good one to use for defining a time standard. The second is know defined as (9 192 631 770) times the period of variation of radiation from the cesium atom.

period is defined as (the time interval needed for one complete vibration.

Atomic clock, uses the characteristic frequency of the cesium 133 atom as the "reference clock".

Customary System

_____________ Another system of units that is still used in the (USA). In this system the units of length, mass and time are the foot (ft), slug and second respectively.

Conversions (length)

1 in = 2.54 cm , 1 m = 39.37 in = 3.281 ft , 1 ft = 0.3048 m ,
1 yd = 3 ft , 1 ft = 12 in , 1 mi = 1.609 km , 1 km = 0.621 mi ,
1 light year = 9.461×10^{15} m

(Mass), (Time)

1 metric ton = 1000 kg , 1 kg = 6.852×10^{-2} slug , 1 slug = 14.59 kg ,
1 year = 365 days = 3.16×10^{7} s , 1 day = 24 hr = 1.44×10^{3} min = 8.64×10^{4} s .

Density and Atomic Mass

The density (ρ) of any substance is defined as its (mass per unit volume) $\rho = m/V$

For example, aluminum has a density of (2.7 g/cm³), lead (11.3 g/cm³).

Atomic mass is defined as the mass of a single atom of the element measured in atomic mass units (u) where: 1u = $1.660\,538\,7 \times 10^{-27}$ kg.

The atomic mass of lead is 207 u and that of aluminum is 27 u.

However the ratio of atomic masses is 207u/27u = 7.67, does not correspond to the ratio of densities $(11.3 \times 10^{3}/2.70 \times 10^{3}) = 4.19$

This discrepancy is due to the difference in atomic spacings and atomic arrangements in the crystal structure of the two elements.

(EX.1)

A solid cube of aluminum ($\rho = 2.7$ g/cm³) has a volume of (0.200 cm³). It is known that (27.0 g) of aluminum contains (6.02×10^{23} atoms). How many aluminum atoms are contained in the cube?

(Solu)

The mass of the cube is: $m = \rho V = (2.7\,\text{g/cm}^3)(0.200\,\text{cm}^3) = 0.540\,\text{g}$.

$$\frac{m_{sample}}{m_{27g}} = \frac{N_{sample}}{N_{27}} \Rightarrow \frac{0.540\,g}{27\,g} = \frac{N_{sample}}{6.02 \times 10^{23}\,\text{atoms}} \Rightarrow N_{sample} = 1.20 \times 10^{22}\ \text{atoms}$$

power	perfix	abbreviation	power	perfix	abbreviation
10^{-24}	yocto	y	10^{-1}	deci	d
10^{-21}	zepto	z	10^{3}	kilo	k
10^{-18}	atto	a	10^{6}	mega	M
10^{-15}	femto	f	10^{9}	giga	G
10^{-12}	pico	p	10^{12}	tera	T
10^{-9}	nano	n	10^{15}	peta	P
10^{-6}	micro	μ	10^{18}	exa	E
10^{-3}	milli	m	10^{21}	zetta	Z
10^{-2}	centi	c	10^{24}	yotta	Y

Dimensional Analysis

We shall often use brackets [ ] to denote the dimensions of a physical quantity, such as [V], [A].
Dimension analysis can be used to check the final expression.
Dimensions can be treated as algebraic quantities.
For example, quantities can be added or subtracted only if they have the same dimensions. Furthermore, the terms on both sides of an equation must have the same dimensions. The relationship can be correct only if the dimensions on both sides of the equation are the same.
. To illustrate this procedure check the validity of ($x=\frac{1}{2}at^2$), an equation of car move with constant acceleration (a) from position (x) at a time (t). The quantity (x) on the left side has the dimension of length. For the equation to be dimensionally correct, the quantity on the right side must also have the dimension of length.
we can perform a dimensional check by substituting the dimensions for acceleration, (L/T^2) and time (T) into the equation

$x=\frac{1}{2}at^2$

$L = \frac{L}{T^2} \cdot T^2 = L$

The dimensions of time cancel as shown. A more general procedure is to use :- $X \propto a^n t^m$

where (n) and (m) are exponents that must be determined and the symbol (X) indicates a proportionality.

$[a^n t^m] = L = L^1 T^0$

$(L/T^2)^n (T)^m = L^1 T^0 \rightarrow (L^n T^{m-2n}) = L^1 T^0$

The exponents of (L) and (T) must be the same on both sides of the equation. We see that $n=1$ and $m-2n=0$ gives us $m=2$, but $(X \propto a^n t^m)$, so that $(X \propto a t^2)$. This result differs by a factor of (1/2) from the correct equation ($X = \frac{1}{2} a t^2$).

System	Area (L^2)	Volume (L^3)	Speed (L/T)	acceleration (L/T^2)
SI	m^2	m^3	m/s	m/s^2
U.S Customary	ft^2	ft^3	ft/s	ft/s^2

Conversion of units

sometimes it is necessary to convert units from one measurement system to another, or to convert within a system.

1 mile = 1609 m = 1.609 km, 1 ft = 0.3048 m = 30.48 cm

1 m = 39.37 in = 3.281 ft , 1 in = 0.0254 m = 2.54 cm

Units can be treated as algebraic quantities that can cancel each other.

EX-2 convert 15.0 in to centimeters.

Solu) $15.0\ in = (15.0\ in)\left(\frac{2.54\ cm}{1\ in}\right) = 38.1\ cm$

EX.3 A car is traveling at a speed of (38.0 m/s). Is this car exceeding the speed limit of (75.0 mil/h)?

Solu) We first convert meters to miles :-

$(38.0\ m/s)\left(\frac{1\ mil}{1609\ m}\right) = 2.36 \times 10^{-2}\ mil/s$

Now we convert seconds to hours

$(2.36 \times 10^{-2}\ mil/s)(60 s/1 min)(60 min/1 h) = 85.0\ mi/h$

Estimates and order of Magnitude Calculations

It is often useful to compute an approximate answer to a given physical problem. Such approximation is usually based on certain assumptions, which must be modified if greater precision is needed. We will sometimes refer to an (order of magnitude) of a certain quantity as the power of ten of the number that describes that quantity. We use the symbol $\sim$ for "is on the order of". Thus:

$0.0087 \sim 10^{-2}$, $0.0021 \sim 10^{-3}$, $740 \sim 10^{3}$

The results are reliable to within about a factor of (10).

Ex. 4 Estimate the number of steps a person would take walking from New York to Los Angeles.

Solu

The distance between these two cities is about 3000 mi.
We can that each step covers about (2 ft) and ($1 mi \simeq 5280 ft$) or ($1 mi \simeq 5000 ft$).

$$\frac{5000\ ft/mi}{2\ ft/step} = 2500\ steps/mi ,$$

$$steps = (3 * 10^{3}\ mi)(2.5 * 10^{3}\ steps/mi) = 7.5 * 10^{6}\ steps \simeq 10^{7}\ steps.$$

Ex. 5

Estimate the number of gallons of gasoline used each year by all the cars in the United States.

Solu

No. of people in the United States $\simeq$ 280 million, No. of cars $\simeq$ 100 million guessing that there are between (2-3) people/car.
Average distance each car travels per year is 10 000 mi, and gasoline consumption of (0.05 gal/mi), then each car uses about (500 gal/yr)
Total consumption is: $500\ gal/year * 100 * 10^{6}\ car = 10^{10} * 5\ gal \simeq 10^{11}\ gal/yr$

Significant Figures

When certain quantities are measured, the measured values are known only to within the limits of the experimental uncertainity. The value of this uncertainity can depend on various factors, such as the ① quality of the apparatus ② the skill of the experimenter, and ③ the number of the measurements performed.

An example of significant figures: suppose that we are asked in a laboratory experiment to measure the area of a computer disk label using a meter stick as a measuring instrument. If the length is measured to be (5.5 cm) and the accuracy to which we can measure the length of the label is (± 0.1 cm).

The length lies between (5.4 cm) and (5.6 cm). If the label width is (6.4 cm), the actual value lies between (6.3 cm) and (6.5 cm). Thus we could write the measured values as (5.5 ± 0.1) cm and (6.4 ± 0.1 cm). The area of the label is ($5.5 * 6.4$) = $35.2\ cm^2$, it is taken as ($35\ cm^2$). This value can range between ($5.4 * 6.3 = 34\ cm^2$) and ($5.6 * 6.5 = 36\ cm^2$).

Rule

"When multiplying several quantities, the number of significant figures in the final answer is the same as the number of significant figures in the quantity having the lowest number of significant figures. The same rule applies to division". Zero may or may not be significant figures.

Rule

"When numbers are added or subtracted, the number of decimal places in the result should equal the smallest number of decimal places of any term in the sum".

General Rule: If the last digit dropped is greater than (5) the last digit retained is to be increased by (1). If the last digit dropped is less than (5), the last digit retained remains as it is.

Examples - Chap.1

ex.1 solve the following to correct significant figures

(a) $9.65+15.237 = 24.887 = 24.89$
(b) $7.24+11.141+0.0025 = 18.3835 = 18.38$
(c) $7.987-5.45 = 2.537 = 2.54$
(d) $29.412-29.4 = 0.012 = 0.0$
(e) $3.8*10^5 - 2.6*10^4 = 3.8*10^5 - 0.26*10^5 = 3.5*10^5 = 3.5*10^5$

ex.2 solve the following to correct significant figures

(a) $210 \div 5.5 = 38.18 = 38$
(b) $5.02*10^3 * 1.81*10^6 \div 0.8926 = 10.179*10^3 = 10.2*10^3$
(c) $4.372*11 = 48.092 = 48$
(d) $6.32*7.4695 = 47.20724 = 47.2$

ex.3
The length, breadth and thickness of a rectangular sheet of metal are 4.234m, 1.005m and 2.01 cm respectively. Give the area and volume of the sheet to correct significant figures.

Solu $l = 4.234$ m, $b = 1.005$ m, $h = 2.01$ cm $= 0.0201$ m
Total area $(S) = 2(lb+bh+lh)$
$S = 2(4.234*1.005+1.005*0.0201+0.0201*4.234)$
$= 8.7209478 = 8.72\ m^2$
Volume $V = l*b*h = 4.234*1.005*0.0201$
$= 0.08552\ m^3$
$= 0.0855\ m^2$

ex.4
Calculate the area of a circle in appropriate number of significant figures if the radius is 0.45m.

Solu

$r = 0.45m$, area of circle $= \pi r^2 = \pi (0.45)^2$
$= 0.63585 = 0.64\ m^2$

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ex.5
The mass of a body is 242.7g and its volume is 24.2cm³. Caclulate the density of the substance upto appropriate significant number.

Solu

$m = 242.7g$, $V = 24.2\ cm^3$, density $(\rho) = m/V$
$\rho = 242.7/24.2 = 10.0289 = 10.0\ g/cm^3$

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ex.6
In an experiment to measure the time period of a simple pendulum, the time period was found to be (2.54, 2.67, 2.72, 2.63 and 2.60) second respectively. Determine

(a) true value of the time period.

sol

$$= \frac{t_1+t_2+t_3+t_4+t_5}{5} = \frac{(2.54+2.67+2.72+2.63+2.60)}{5}$$
$= 2.63\ s$

(b) Absolute value of errors in different measurements

$\Delta T_1 = (2.63-2.54) = 0.09s$, $\Delta T_2 = (2.63-2.67 = 0.04s$
$\Delta T_3 = (2.63-2.72) = 0.09s$, $\Delta T_4 = (2.63-2.63) = 0.0s$
$\Delta T_5 = (2.63-2.60) = 0.03s$

Mean absolute error $= \Delta T = \frac{(0.09+0.04+0.09+0.0+0.03)}{5}$
$= 0.05\ sec$

(c) Relative error $= \Delta T/T = 0.05/2.63 = 0.019 = 0.02$

(d) percentage error $= 0.02 \times 100 = 2\%$

ex.7

The resistance of a conductor is given by $R = \frac{V}{I}$, where $V = (200 \pm 5)V$, and $I = (20 \pm 0.4)A$. Calculate total error in R.

Solu $R = V/I$, $\Delta R/R = \Delta V/V + \Delta I/I$

percentage error in R is $\frac{\Delta R}{R} * 100 = \frac{\Delta V}{V} * 100 + \frac{\Delta I}{I} * 100$

$$\Delta R/R * 100 = \left(\frac{5}{200} * 100\right) + \left(\frac{0.4}{20} * 100\right)$$
$$= (2.5 + 2)\% = 4.5\%$$

ex-8

The side of a cube is measured to be (5.5 ± 0.1) cm. Find the volume of the cube.

Solu

$$a = 5.5 \pm 0.1 \text{ cm}, \quad V = a^3 = (5.5)^3 = 166.375 = 166.4 \text{ cm}^3$$

$$\frac{\Delta V}{V} = 3\frac{\Delta a}{a} \Rightarrow \Delta V = 3V\frac{\Delta a}{a} = 3 * \frac{0.1}{5.5} * 166.4 = 9.076 = 9.1 \text{ cm}^3$$

$$\therefore V = (166.4 \pm 9.1) \text{ cm}^3$$

ex.9

A capacitor of the capacitance is $C = (2.0 \pm 0.1)$ MF charged to a voltage (20 ± 0.2)V. What is the charge on the capacitor?

solu

$$Q = CV = (2.0 * 10^{-6}) * 20 = 40.0 * 10^{-6} C$$

$$\frac{\Delta Q}{Q} = \frac{\Delta C}{C} + \frac{\Delta V}{V} \Rightarrow \Delta Q = \left(\frac{0.1}{2.0} + \frac{0.2}{20}\right) * 40 * 10^{-6}$$

$$\Delta Q = 2.4 * 10^{-6} C$$

$$\therefore Q = (40.0 \pm 2.4) * 10^{-6} C$$

ex. 10

The distance covered by a body in time (5.0± 0.6) is (40.0±0.4) m
Calculate the speed of the body. Also determine the percentage error in the speed.

(Solu) $S = (40.0 \pm 0.4)\,m, \quad t = (5.0 \pm 0.6)\,s$

$$V = S/t = 40.0/5.0 = 8.0\,m\,s^{-1}.$$

$$\frac{\Delta V}{V} = \frac{\Delta S}{S} + \frac{\Delta t}{t} \Rightarrow \Delta V = \left(\frac{0.4}{40} + \frac{0.6}{5.0}\right) * 8.0 = 1.04$$

speed $V = (8.0 \pm 1.04)\,m/s$

$$\%\,\text{error} = \frac{\Delta V}{V} * 100 = \frac{1.04}{8} * 100 = 13\%$$

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ex. 11

The length and breadth of a field are measured as

$$l = (100 \pm 2)\,m \quad , b = (80 \pm 1)\,m$$

What is the area of the field.

(Solu)

$$A = l * b = 100 * 80 = 8000\,m^2$$

$$\frac{\Delta A}{A} = \frac{\Delta l}{l} + \frac{\Delta b}{b}$$

$$\Delta A = \left(\frac{2}{100} + \frac{1}{80}\right) * 8000 = 260\,m^2$$

Area of the field $= A \pm \Delta A$

$$= (8.0 \pm 0.26) \times 10^3\,m^2$$

---

ex. 12

An ore loader moves (1200 tons / hr) from a mine to the surface. Convert this rate to (lb/s) using (1 ton = 2000 lb)

(Solu)

$$\text{Rate} = \frac{(1200\ \text{ton}) * (2000\ \text{lb/ton})}{(1\ \text{hr}) * (3600\ \text{sec/hr})} = 666.67\ \text{lb/sec}$$

ex.13
Two spheres are cut from a certain uniform rock. One has radius (5 cm). The mass of the other is (6) times greater. Find its radius.

Solu
$\rho_1 = \rho_2 \Rightarrow \frac{m_1}{V_1} = \frac{m_2}{V_2}$, but $m_2 = 6m_1$, $\therefore \frac{m_1}{V_1} = \frac{6 m_1}{V_2}$

or $V_2 = 6V_1$, $V = \frac{4}{3}\pi r^3 \Rightarrow r_2^3 = 6r_1^3$ or $r_2 = \sqrt[3]{6r_1^3}$

$r_2 = \sqrt[3]{6(5\,cm)^3} \Rightarrow r_2 = 9.08\,cm$

ex.14
Find the order of magnitude of your age in seconds.

solu suppose your age is (20 year + 4 months + 20 day).
Age = $(20 \times 365 \times 24 \times 3600 + 4 \times 30 \times 24 \times 3600 + 20 \times 24 \times 3600) = 642816000 \simeq 6.428 \times 10^8$ second

ex.15
Estimate the number of human hearth beats in an average life time (70 years). Take 72 beats/min.

Solu
No. of beats $= (70 \times 365 \times 24 \times 60 \times 72) = 2.65 \times 10^9$

ex.16
A worker is to paint the walls of a square room (8 ft) high and (12 ft) along each side

13 1311 Chap.

1 What surface area in square meters must be cover. Take 1m = 3.28 ft

Solu)

$$\text{surface area } (m^2) = \left[(12\times8)ft^2/(3.28)^2\right]m^2 \times 4 = 35.69\, m^2$$

ex. 17

A car is travelling at a speed of (125 ft/s). Is this car exceeding the speed limit of (120 km/hr)? Take 1m = 3.28 ft, 1 km = 10^3 m

Solu)

$$125\ ft/s = \left(\frac{125/10^3\ km}{3.28}\right)/hr = 137.15\ km/h > 120$$

ex. 18

A car is travelling at a speed of (38 m/s). Is this car exceeding the speed limit of (75 mil/hr)? Take 1 mil = 1.609 km

Soly)

$$38\ m/s = \left[\left(\frac{38\times3600\ km/hr}{1000}\right)/1.609\right] = 85.02\ mil/hr$$

ex. 19

One gallon of paint (volume = $3.78\times10^{-3}\ m^3$) covers an area of (25 m²) what is the thickness of the paint on the wall?

Solu)

$$t = \frac{V}{A} = \left(\frac{3.78\times10^{-3}\ m^3}{25\ m^2}\right) = 0.1512\times10^{-3}\ m = 0.1512\ mm$$

ex. 20

What are the dimensions of (K) in the following Bernoulli's equation:

$$\frac{P}{\rho} + gh + \frac{V^2}{2} = K$$

Solu

Dim of $\frac{P}{\rho}$ = dim. of gh = dimen. of V^2

Dim. of K = Dim of $V^2 = (LT^{-1})^2 = L^2T^{-2}$

ex.21

A physical quantity (p) is related to four variables (a, b, c and d) as follows ($p = a^3 b^2 / \sqrt{c}\, d$). The percentage error in the measurement in a, b, c and d are (1%, 3%, 4% and 2%) respectively. What is the percentage error in the quantity (p)?

Solu $p = \frac{a^3 b^2}{\sqrt{c}\, d}$, $\frac{\Delta p}{p} = 3\frac{\Delta a}{a} + 2\frac{\Delta b}{b} + \frac{1}{2}\frac{\Delta c}{c} + \frac{\Delta d}{d}$

$\frac{\Delta P}{P} \times 100 = 3(\frac{\Delta a}{a}) \times 100 + 2(\frac{\Delta b}{b} \times 100) + \frac{1}{2}(\frac{\Delta c}{c}) \times 100 + (\frac{\Delta d}{d}) \times 100$

$= 3 \times 1\% + 2 \times (3\%) + \frac{1}{2}(4\%) + (2\%)$

$= 13\%$

ex.22

Using the principle of homogeneity of dimensions check the accuracy of following equation.

$W = mgh + \frac{1}{2} m V^2$

Solu

$W = mgh + \frac{1}{2} mV^2$

$N.m = Kg . \frac{m}{s^2} . m + \frac{1}{2} Kg \frac{m^2}{s^2}$

$N.m = N.m + \frac{1}{2} N.m$

where $N = Kg . m/s^2$

ex.23

A potential difference $V = (80 \pm 2)V$ is applied across a conductor. The current in the conductor is $(10 \pm 0.5)A$. Calculate the error in the measurement of resistance of conductor.

Solu

$$R = \frac{V}{I} = \frac{80}{10} = 8\,\Omega$$

$$\frac{\Delta R}{R} = \frac{\Delta V}{V} + \frac{\Delta I}{I} \Rightarrow \Delta R = \left(\frac{2}{80} + \frac{0.5}{10}\right)8 = 0.6\,\Omega$$

ex 24

What is the volume of a sphere with radius (12.4 cm).

Solu

$$V = \frac{4}{3}\pi r^3 = \frac{4}{3}\pi(12.4)^3 = 7.98 \simeq 8\,cm^3$$

Lecture

(2)

Chap. 2
Motion in One Direction

Particle model :

A particle is "a point-like object that is an object with mass but having infinitesimal size".

For example : to describe the motion of the Earth around the Sun, we can treat the Earth as a particle and obtain reasonably accurate data about its orbit.

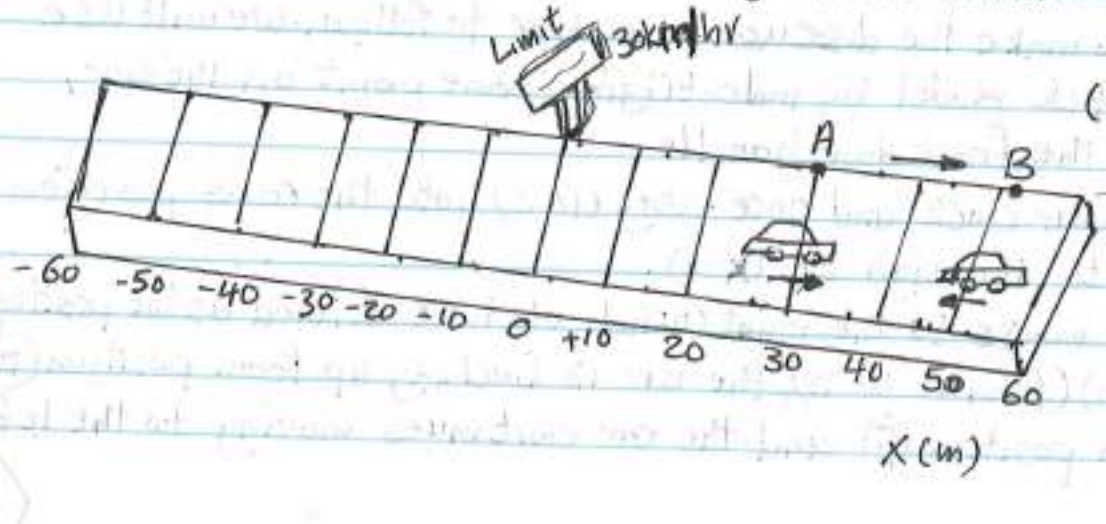
This approximation is justified because the radius of the Earth's orbit is large compared with the dimensions of the Earth and the Sun.

As an example on a much smaller scale, it is possible to explain the pressure exerted by a gas on the walls of a container by treating the gas molecules as particles, without regard for the internal structure of the molecules.

To describe the moving object as a particle we can use what is called the (particle model).

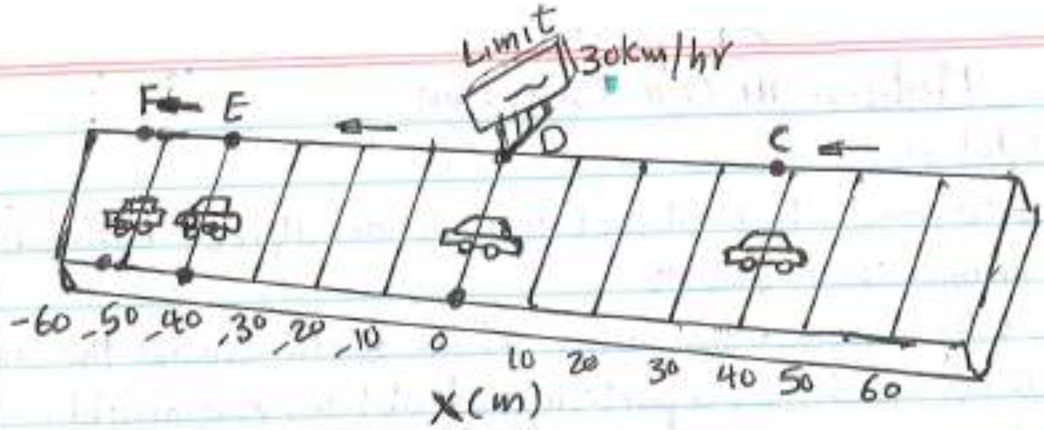
Position, velocity and speed

The motion of a particle is completely known if the particles position in space is known at all times. A particle's position is "the location of the particle with respect to chosen reference point that we can consider to be the origin of a coordinate system.



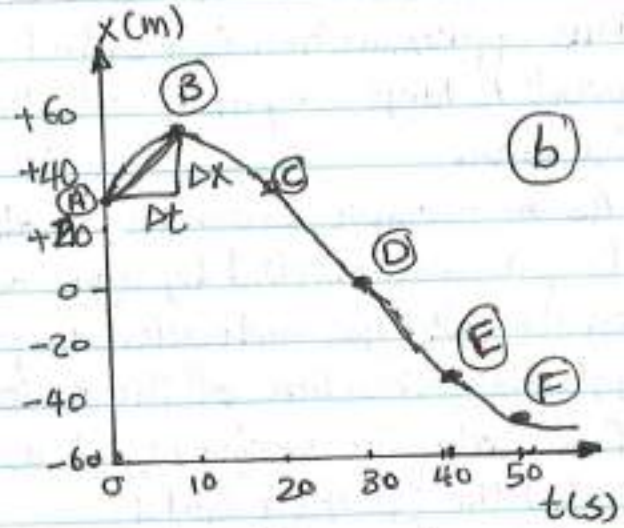
(Fig 2.1)

(a)



position of the car at various time

position	t(s)	X(m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	-37
F	50	-53



consider car moving back and forth along the x axis as in figure (2.1/a). The car is at (30m) to the right of a road sign which we will use to identify the reference position ($X=0$). (Assume all data are known to two significant figures). The initial position is (3.0×10m) it is written in the simpler form (30m) to make the discussion easier to follow. we will use the particle model by indentifying some point on the car, perhaps the front door handle.

we start our clock and once every (10 s) note the car's position relative to the sign at ($X=0$).

The car moves to the right (which we have defined as the positive direction) (from A to B), the car is backing up from position (B) through position (F) and the car continues moving to the left

and is more than (50m) to the left of the sign when we stop recording information is as shown in figure. It is called (a position-time graph).
The displacement of a particle is defined as "its change in position in some time interval".

$$\Delta x \equiv x_f - x_i$$

Δx = displacement or change in position of the particle.
x_i ≡ initial position , x_f = final position
If $x_f > x_i$, Δx is positive, if $x_f < x_i$; Δx is negative
Displacement differ from distance that (distance is length of a path followed by a particle).
Displacement is an example of a vector quantity. Position, velocity and acceleration also are vectors. In general a vector quantity requires the specification of both direction and magnitude. By contrast, a scalar quantity has a numerical value and no direction.
Distance is a scalar quantity and it is always represented as a positive number while displacement can be either positive or negative.
The average velocity ($\bar{V}_x$) of a particle is defined as (the particle's displacement Δx divided by the time interval Δt during which that displacement occurs).

$$\bar{V}_x = \frac{\Delta x}{\Delta t}$$

The unite of ($\bar{V}_x$) is (m/s) in SI units.
The average velocity of a particle moving in one direction can be positive or negative depending on the sign of the displacement. The time interval (Δt) is always positive. The slope of the line between the points Ⓐ and Ⓑ on the position-time graph in (fig 2.1/b) represents the ratio ($\Delta x/\Delta t$) which is defined as (average velocity).

e.g: the average velocity of the car between points (A) and (B) is:

$$\bar{V}_x = \frac{(52-30)m}{(10-0)s} = 2.2 \ m/s$$

The terms (speed) and (velocity) are interchangeable. In physics however, there is a clear distinction between these two quantities. The (average speed) of a particle, a scalar quantity, is defined as (the total distance traveled divided by the total time interval required to travel that distance).

Average speed = total distance / total time

Unlike average velocity, average speed has no direction and hence carrying no algebraic sign, they have the same units and the magnitude of the average velocity is not the average speed.

e.g.1

Find the displacement, average velocity and average speed of the car in (fig 2.1/a) between points (A) and (F).

Solu

From the position-time graph given in (fig 2.1/b) note that ($x_A = 30m$) at ($t_A = 0$), and that ($x_F = -53m$) at ($t_F = 50s$).

$\Delta x = -53m - 30m = -83m$ displacement.

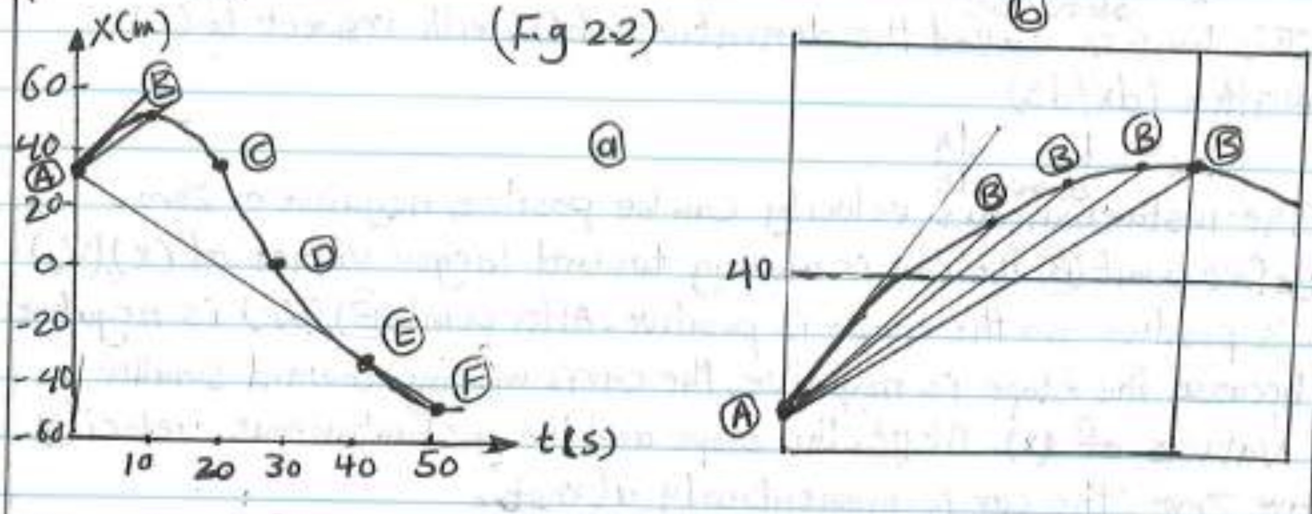
$$\bar{V}_x = \Delta x / \Delta t = \frac{x_f - x_i}{t_f - t_i} = \frac{x_F - x_A}{t_F - t_A} = \frac{(-53-30)m}{(50-0)s} = \frac{-83m}{50s} = -1.7 \ m/s.$$

To calculate the average speed we need the complete details about the distance between the points [from (A) to (B) is (22m) plus from (B) to (F) for a total of 127m]

average speed = 127m / 50s = 2.5 m/s

Instantaneous velocity and speed

It is the velocity of a particle at a particular instant in time. Consider (fig 2.2/a) which is a reproduction of the graph in (fig 2.1/b). The slope of the line (AB) represents the average velocity for the interval during which the car moved from position (A) to position (B).



The slope of the line (AF) represents the average velocity of the interval during which it moved from (A) to (F). Which of these two lines do you think is a closer approximation of the initial velocity of the car?

The car starts out by moving to the right (positive direction), the value of the average velocity during the (A) to (B) interval is more representative of the initial value than is the value of the average velocity during the (A) to (F) interval, which is negative (eg1).

Now let us focus on the line (AB) and slide point (B) to the left along the curve toward point (A) as in (Fig 2.2/b). The line between points becomes steeper and steeper and as the two points become extremely close together, the line becomes a tangent line to the curve. The slope of this tangent line represents the velocity of the car at the moment we started taking data, at point (A). The instantaneous velocity (V_x) equals: "the limiting value of the ratio ($\Delta X/\Delta t$) as (Δt) approaches zero".

[Note: the displacement (ΔX) also approaches zero as (Δt) approaches zero, so that the ratio looks like 0/0. As (ΔX) and (Δt) become smaller and smaller, the ratio ($\Delta X/\Delta t$) approaches a value equal to the slope of the line tangent to the (x) versus (t) curve].

$$V_x = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t}$$

This limit is called the derivative of (x) with respect to (t), written (dx/dt)

$$V_x \equiv \lim_{\Delta t \to 0} \frac{dx}{dt}$$

The instantaneous velocity can be positive, negative or zero. Before point Ⓑ the car is moving toward larger values of (x), (V_x) is positive so the slope is positive. After point Ⓑ, (V_x) is negative because the slope is negative, the car is moving toward smaller values of (x). At Ⓑ, the slope and the instantaneous velocity are zero, the car is momentarily at rest.

The instantaneous speed of a particle is defined as the magnitude of its instantaneous velocity. It has no direction and hence carries no algebraic sign.

For example: if one particle has an instantaneous velocity of (+35 m/s) along a given line and another particle has an instantaneous velocity of (-35 m/s) along the same line, both have a speed of (35 m/s).

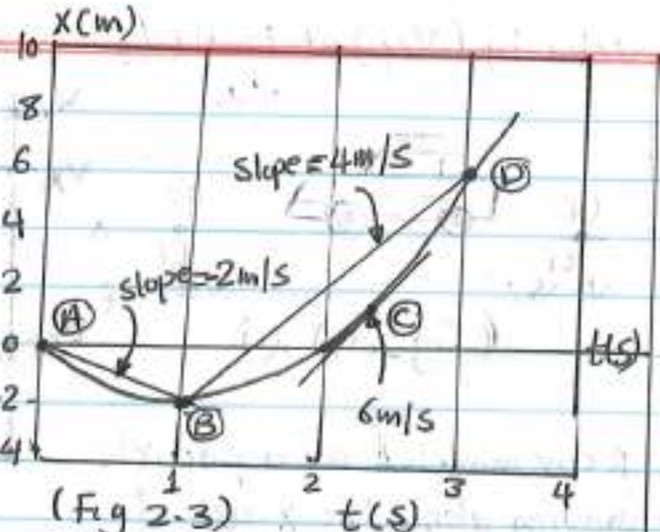
We can use the word velocity to designate instantaneous velocity,
" " " " " speed " " " speed.

e.g/2

A particle moves along the x axis. Its position varies with time according to the expression ($x = -4t + 2t^2$) where (x) is in meters and (t) is in seconds. The position-time graph is shown in (fig 2.3). Note that the particle moves in the negative (x) direction for the first second of motion, is momentarily at rest at the moment (t = 1s) and moves in the positive direction at times (t > 1s).

ⓐ Determine the displacement of the particle in the time intervals (t = 0) to (t = 1s) and (t = 1s to t = 3s).

Soly:



During the first time interval, the slope is negative and hence the average Velocity is negative. Thus, we known that the displacement between (A) and (B) must be negative. The displacement between (B) and (D) is positive. $t_i = t_A = 0$,

(Fig 2-3) t(s)

$t_f = t_B = 1s$ and $x = -4t + 2t^2$.

$$\Delta x_{A \to B} = x_f - x_i = x_B - x_A = [-4(1) + 2(1)^2] - [-4(0) + 2(0)^2]$$
$= -2m$ between $t = 0$ and $t = 1s$.

Between $t = 1s$ and $t = 3s$, $t_i = t_B = 1s$ and $t_f = t_D = 3s$.

$$\Delta x_{B \to D} = x_f - x_i = x_D - x_B = [-4(3) + 2(3)^2] - [-4(1) + 2(1)^2]$$
$= +8m$

(b) Calculate the average velocity during these two time intervals

In the first time interval $\bar{v}_{x(A \to B)} = \frac{\Delta x_{A \to B}}{\Delta t} = \frac{x_B - x_A}{t_B - t_A} = \frac{-2m}{1s} = -2\, m/s$.

In the second time interval $\Delta t = t_D - t_B = 3 - 1 = 2s$

$$\bar{v}_{x(B \to D)} = \frac{\Delta x_{B \to D}}{\Delta t} = 8m/2s = 4\, m/s$$

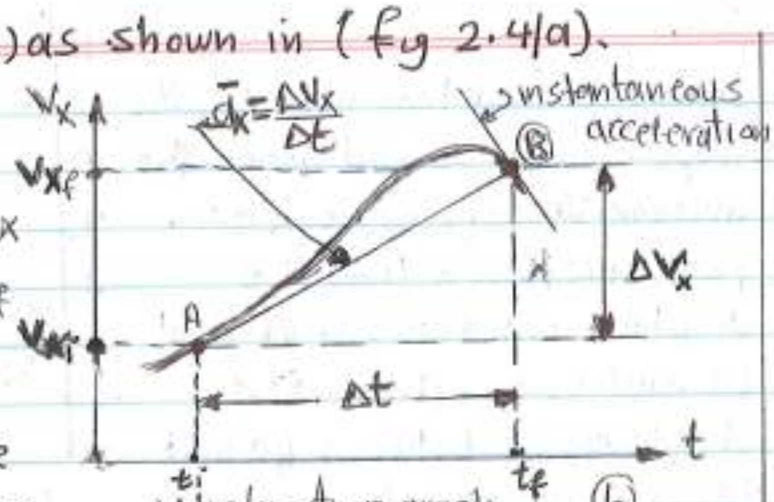
(C) Find the instantaneous Velocity of the particle at ($t = 2.5s$).

Soly By measuring the slope of the line at ($t = 2.5s$) we find that $v_x = +6\, m/s$

Acceleration

when the velocity of a particle changes with time the particle is said to be accelerating. How to quantify acceleration? suppose an object that can be modeled as a particle moving along the x axis has an initial velocity (v_{xi}) at time (t_i) and a final velocity (V_{xf}) at time (t_f) as shown in (fig 2.4/a).

(Fig 2.4) ⓐ



velocity - time graph ⓑ

A car modeled as a particle moving along the x-axis from Ⓐ to Ⓑ. $\Delta t = t_f - t_i$, the slope of the line in (fig 2.4/b) is the average acceleration ($\Delta V_x = V_{xf} - V_{xi}$).

The average acceleration ($\bar{a}_x$) of the particle is defined as (the change in velocity ΔV_x divided by the time interval Δt during which that change occurs).

$$\bar{a}_x = \frac{\Delta V_x}{\Delta t} = \frac{V_{xf} - V_{xi}}{t_f - t_i}$$

The SI unit of acceleration is (m/s^2).

The instantaneous acceleration is defined as "the limit of the average acceleration as (Δt) approach (Zero). It is equal to :-

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta V_x}{\Delta t} = \frac{dV_x}{dt}$$

That is, the instantaneous acceleration equals" the derivative of the velocity with respect to time. It is the slope of the velocity_time graph (the slope of the line at point Ⓑ in fig 2.4/b).

If (a_x) is positive, the acceleration is in the positive (x) direction.

If (a_x) is negative, the acceleration is in the negative (x) direction

Negative acceleration does not necessarly mean that an object is slowing down. If the acceleration is negative and the velocity is negative, the object is speeding up.

when the object's velocity and acceleration are in the same direction, the object is speeding up. On the other hand, when the object's velocity and acceleration are in opposite directions, the object is slowing down.'

The acceleration is caused by force exerted on the object.

$$F \propto a$$

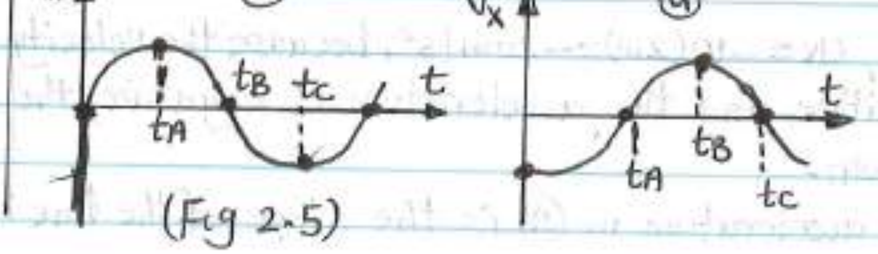
force and acceleration are both vectors and the vectors act in the same direction. It is very useful to equate the direction of the acceleration to the direction of a force. We use the term (acceleration) to mean (instantaneous acceleration), and when we use (average) we mean (average acceleration).

The acceleration can also be written:-

$$a_x = \frac{dv_x}{dt} = \frac{d}{dt}\left(\frac{dx}{dt}\right) = \frac{d^2x}{dt^2}$$

That is in one-dimensional motion, the acceleration equals the second derivative of (x) with respect to time.

The acceleration at any time is the slope of the velocity-time graph at that time. positive values of acceleration corresponds to those points in (fig 2.5/a) where the velocity is increasing in the positive (x) direction. The acceleration reaches a maximum at time (t_A), when the slope of the (velocity-time graph) is a maximum. The acceleration then goes to zero at time (t_B), when the velocity is maximum (slope of $v_x - t$ graph) is zero. The acceleration is negative when the velocity is decreasing in the positive (x) direction.

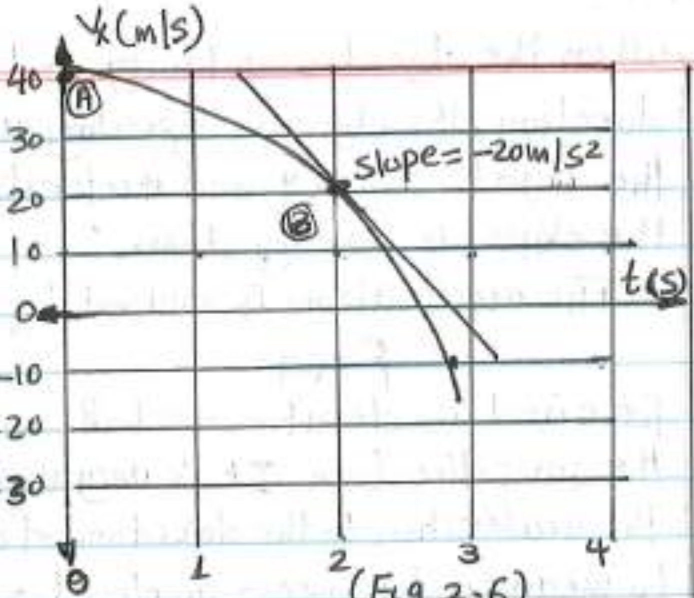


(Fig 2.5)

The instantaneous acceleration can be obtained from the velocity-time graph (a) e.g

The velocity of a particle moving along the axis varies in time according to the expression $(v_x = 40-5t^2)$ m/s, where (t) is in seconds.

(A) Find the average acceleration in the time interval :-
$t=0$ to $t=2.0$ s



(Fig 2-6)

Solu

$t_i = t_A = 0$ and $t_f = t_B = 2.0$ s

$v_{xA} = (40-5t_A^2) = [40-5(0)^2]$ m/s $= +40$ m/s

$v_{xB} = (40-5t_B^2) = [40-5(2.0)^2] = +20$ m/s

The average acceleration in the specified time interval $\Delta t = t_B - t_A = 2.0 - 0.0 = 2.0$ s is :-

$$\bar{a}_x = \frac{v_{xf}-v_{xi}}{t_f - t_i} = \frac{v_{xB}-v_{xA}}{t_B - t_A} = \frac{(20-40)}{(2.0-0.0)} = -10 \text{ m/s}^2$$

(B) Determine the acceleration at $(t = 2.0$ s$)$.

Solu

The velocity at any time (t) is $(v_{xi} = 40-5t^2)$ m/s and the velocity at any later time $(t+\Delta t)$ is

$$v_{xf} = 40-5(t+\Delta t)^2 = 40-5t^2 - 10t\,\Delta t - 5\,\Delta t^2$$

The change in velocity over the time interval (Δt) is :-

$$\Delta v_x = v_{xf} - v_{xi} = (-10t\,\Delta t - 5\Delta t^2) \text{ m/s}$$

$$a_x = \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \lim_{\Delta t \to 0} (-10t - 5\Delta t) = -10t \text{ m/s}^2$$

Therefore at $t = 2.0$ s, $a_x = -10(2.0) = -20$ m/s², because the velocity of the particle is positive and the acceleration is negative, the particle is slowing down.

(Note): the average acceleration in (A) is the slope of the line connecting points (A) and (B). The instantaneous acceleration in (B) is the slope of the line tangent to the curve at point (B), the acceleration in this example is not constant.

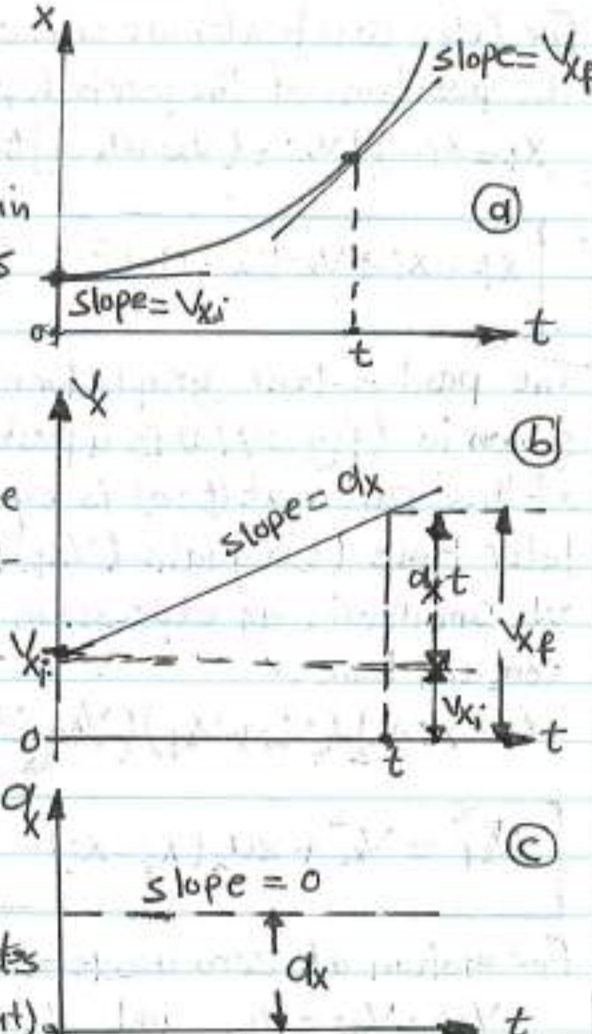
One-Dimensional Motion With Constant Acceleration

In this case, the average acceleration ($\bar{a}_x$) over any time interval is numerically equal to the instantaneous acceleration (a_x) at any instant within the interval and the velocity changes at the same rate throughout the motion.

If we replace ($\bar{a}_x$) by (a_x) and take ($t_i = 0$) and ($t_f = t$) by any later time (t), the equ. $\bar{a}_x = \frac{v_{xf} - v_{xi}}{t_f - t_i}$ becomes:-

$$a_x = \frac{v_{xf} - v_{xi}}{t - 0} \quad \text{(or)}$$

$$\boxed{v_{xf} = v_{xi} + a_x t}$$ powerful expression



(Fig 2.7) A particle moving along x axis with constant acceleration a_x

The powerful expression enables us to determine an object's velocity at any time (t) if we know the object's initial velocity (v_{xi}) and its (constant) acceleration (a_x).

In (fig 2.7/b) (velocity-time graph), the slope of the straight line is the constant slope ($a_x = dv_x/dt$). When the acceleration is constant, the graph of acceleration-time is a straight line having slope of zero.

The average velocity in any time interval at constant acceleration

(a_x) is : $\bar{V}_x = \frac{V_{xi}+V_{xf}}{2}$ It is applied only for $(a_x = \text{constant})$.

The position of an object as a function of time when $(\Delta x = x_f - x_i)$ and $(\Delta t = t_f - t_i = t - 0 = t)$ is:-

$$x_f - x_i = \bar{V}_x t = \frac{1}{2}(V_{xi}+V_{xf})t \Rightarrow \boxed{x_f = x_i + \frac{1}{2}(V_{xi}+V_{xf})t}$$

For $(a_x = \text{constant})$ we can obtain another useful expression for the position of the particle, moving with constant acceleration.

$$x_f = x_i + \frac{1}{2}[V_{xi} + (V_{xi} + a_x t)]t$$

$$\boxed{x_f = x_i + V_{xi}t + \frac{1}{2}a_x t^2} \quad \text{for } (a_x = \text{constant})$$

The position-time graph for motion at constant (positive) acceleration shown in (fig 2-7/a) is a prabola, the slope of the tangent line of this curve at $(t=0)$ is equal to (V_{xi}) and the slope at any later time (t) equals (V_{xf}).

We can obtain an expression for the final velocity that does not contain time:-

$$x_f = x_i + \frac{1}{2}(V_{xi}+V_{xf})\left(\frac{V_{xf}-V_{xi}}{a_x}\right) = \frac{V_{xf}^2 - V_{xi}^2}{2a_x} + x_i$$

$$\boxed{V_{xf}^2 = V_{xi}^2 + 2a_x(x_f - x_i)} \quad \text{for } (a_x = \text{constant})$$

For motion at zero acceleration, we see:-

$$V_{xf} = V_{xi} = V_x \quad \text{and} \quad x_f = x_i + V_x t \quad \text{when } a_x = 0$$

That is when $(a_x = 0)$, the velocity of the particle is constant and its position changes linearly with time.

[These relationships are (Kinematic equations) that may be used to solve any problem involving one-dimensional motion at constant acceleration].

Note: for these equations the motion is along x axis-

e.g. A car traveling at a constant speed of (45 m/s) passes a trooper hidden behind a billboard. One second after the speeding car passes the billboard, the trooper sets out from the billboard to catch it, accelerating at a constant rate of ($3 m/s^2$). How long does it take her to overtake the car?

Solu

First, we write expressions for the position of each vehicle as a function of time. It is convenient to choose the position of the billboard as the origin and to set ($t_B=0$) as the time the trooper begins moving. At that instant, the car has already traveled a distance of (45.0 m) because it has traveled at a constant speed of ($V_x=45.0 m/s$) for 1s ($t_A=-1.0 s$). Thus the initial position of the speeding car is ($X_B=45.0 m$).

Because the car moves with constant speed, its acceleration is zero. The car's position at any time (t) for ($a_x=0$) is:-

$$X_{car}=X_B+V_{x\,car}t=45.0 m+(45.0 m/s)t$$

At ($t=0$), the initial position of the car when the trooper begins to move is: $X_{car}=X_B=45.0 m$

The position of the trooper at any time (t) is: $X_f=X_i+V_{xi}t+\frac{1}{2}a_x t^2$

$$X_{trooper}=0+(0)t+\frac{1}{2}(3.00 m/s^2)t^2$$

At the instant when the trooper position matches that of the car:-

$$X_{trooper}=X_{car}$$

$$\frac{1}{2}(3.00 m/s^2)t^2=45.0 m+(45.0 m/s)t \Rightarrow 1.50t^2-45.0t-45.0=0$$

The positive solution of this equation is $t=31.0 s$.

When the trooper had a more powerful motorcycle with a large acceleration, the time at which the trooper catches the car will be less:-

$$\frac{1}{2}a_x t^2-V_{x\,car}t-X_B=0 \Rightarrow t=-\frac{V_{x\,car}\pm\sqrt{V_{x\,car}^2+2a_xX_B}}{a_x}$$

$$t=\frac{-V_{x\,car}}{a_x}\pm\sqrt{\frac{V_{x\,car}^2}{a_x^2}+\frac{2X_B}{a_x}}$$

Freely Falling Objects

A freely falling object is "any object moving freely under the influence of gravity alone, regardless of its initial motion. Objects thrown upward or downward and those released from rest are all falling once freely they are released. Any freely falling object experiences an acceleration directed downward, regardless of its initial motion.

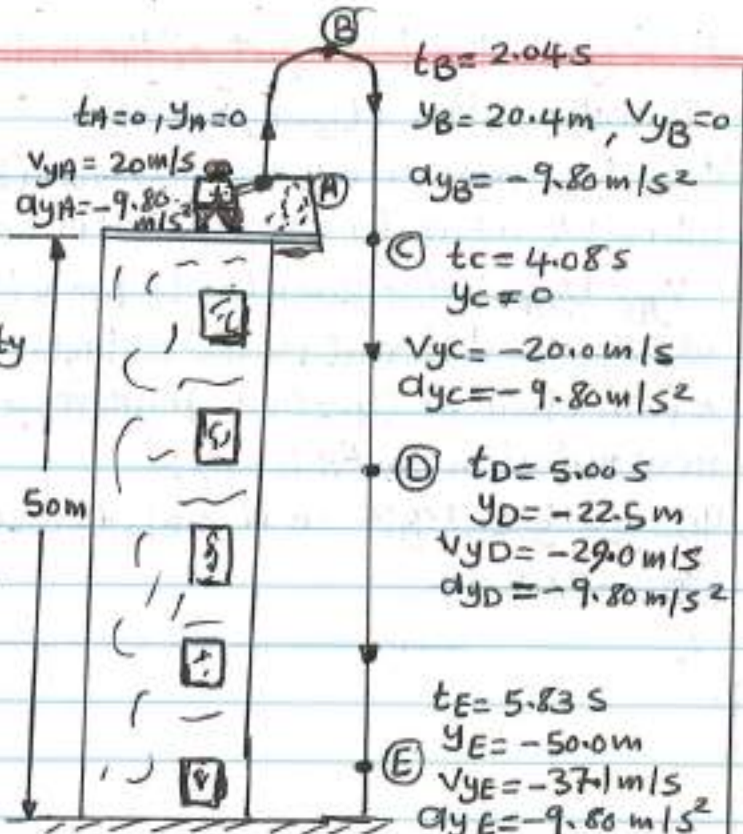
The magnitude of the (free-fall acceleration) by the symbol (g). The value of (g) near the Earth's surface decreases with increasing altitude. At the Earth's surface, the value of (g) is $\simeq 9.80\, m/s^2$. For making quick estimates, use ($g = 10\, m/s^2$). If we neglect air resistance and assume that the free-fall acceleration does not vary with altitude over short vertical distances, then the motion of a freely falling object moving vertically is equivalent to motion in one direction under constant acceleration.

We can use the (Kinematic equations) with the modification that the motion is in the vertical direction (y-direction) rather than in the horizontal direction (X) and the acceleration is downward and equal ($9.80 m/s^2$) we choose ($a_g = -9.80\, m/s^2$). The negative sign means that the acceleration of a freely falling object is downward.

e.g

A stone thrown from a top of a building is given an initial velocity of ($20.0\, m/s$) straight upward. The building is ($50.0\, m$) high, and the stone just misses the edge of the roof on its way down as shown in (fig 2.8). Using ($t_A = 0$) as the time the stone leaves the thrower's hand at position (A), determine (a) the time at which the stone reaches its maximum height, (b) the maximum height, (c) the time at which the stone returns to the height from was thrown, (d) the velocity of the stone at this instant, and (e) the velocity and position of the stone at ($t = 5.00\,s$).

(Solu)



ⓐ as the stone travels from Ⓐ to Ⓑ its velocity must change by (20m/s) because it stops at Ⓑ. Because gravity causes vertical velocities to change by about (10m/s) for every second of free fall, it should take the stone about (2s) to go from Ⓐ to Ⓑ in our drawing. To calculate the exact time we use

$V_{yB} = V_{yA} + a_y t$, $V_{yB} = 0$, $t_A = 0$

$0 = 20.0\,m/s + (-9.80\,m/s^2)\,t \Rightarrow t = t_B = 20.m/s / 9.80\,m/s^2$

$t_B = 2.04\,s$

ⓑ Because the average velocity for this first interval is $\left(\frac{20\,m/s + 0\,m/s}{2} = 10\,m/s\right)$ and because it travels for about (2s), we expect the stone to travel about (20m).

$y_{max} = y_B = y_A + V_{yA} t + \frac{1}{2} a_y t^2 = 0 + (20\,m/s)(2.04\,s) + \frac{1}{2}(9.80\,m/s^2)(2.04\,s)^2$

$= 20.4\,m$

ⓒ The time needed for the stone to go from Ⓐ to Ⓒ should be twice the time needed for it to go from Ⓐ to Ⓑ.

$y_C = y_A + V_{yA} t + \frac{1}{2} a_y t^2 \Rightarrow 0 = 0 + 20.0\,t + \frac{1}{2}(-9.80\,m/s^2)\,t^2$

There is two solution, $t = 0$ corresponding to the time the stone starts its motion, and $t = 4.08\,s$. Notice that $t_C = 2t_B$.

ⓓ $V_{yC} = V_{yA} + a_y t = 20.0\,m/s + (-9.80\,m/s^2)(4.08\,s) = -20.0\,m/s$.

The velocity of the stone when it arrives back at its original height is equal in magnitude to its initial velocity but opposite in direction.

(e) We ignore the first part of the motion (A) → (B). Point (B) is the initial position ($t_B = 0$, $v_{yB} = 0$).

$v_{yD} = v_{yB} + a_y t = 0\,m/s + (-9.80\,m/s^2)(5.00 - 2.04\,s) = -29.0\,m/s$

when we return to our original initial time ($t_A = 0$):-

$v_{yD} = v_{yA} + a_y t = 20.0\,m/s + (-9.80\,m/s^2)(5.00\,s) = -29.0\,m/s$.

We can use different initial instants of time to find the position of the stone at ($t_D = 5.00\,s$) with respect to ($t_A = 0$) by defining a new initial instant ($t_C = 0$):-

$y_D = y_C + v_{yC} t + \frac{1}{2} a_y t^2 = 0 + (-20\,m/s)(5.00 - 4.08)s + \frac{1}{2}(9.80\,m/s^2)(5.00 - 4.08)^2$

$y_D = -22.5\,m$.

Lecture

(3)

ex.1

Calculate the distance of sun from Earth if light take (8) minute to reach Earth from the surface of sun.

Solu

Distance = velocity * time = $(3\times10^8 m/s)(8\times60 s)$
$= 1.44\times10^{11} m$

ex.2

A bomb is dropped from aeroplane when it is directly above a target at a height (2000m). The aeroplane is moving horizontally with a speed of (360 km/h). By how much distance will the bomb miss the target.
Take: $g = 10 m/s^2$

Solu

$v = 360 km/h = 360\times1000/3600 = 100 m/s$
$h_y = y_f = y_i + v_{yi}t + \frac{1}{2}a_yt^2$
$2000 = 0 + 0 + \frac{1}{2}(10)t^2 \rightarrow t = 20 sec$
distance = velocity * time = $100 m/s \times 20 s = 2000 m$

ex.3

A boy on a cliff (49 m) high drops a stone. After one second he throws a second stone. The both hit the ground at the same time. With what speed he throw the second stone?

Solu

For First stone $v_{yi} = 0$, $y_i = 0$
$y_f = y_i + v_{yi}t + \frac{1}{2}gt^2 \rightarrow 49 = 0 + \frac{1}{2}(10 m/s^2)t^2$
$t = 3.16 sec$
For second stone

$t = (3.16 - 1) = 2.16\ s$

$y_f = y_1 + v_{y_1} t + \frac{1}{2} g (t-1)^2$

$49 = 0 + v_{y_1}(2.16) + \frac{1}{2}(10)(3.16-1)^2$

$v_{y_1} = 12.1\ m/s$

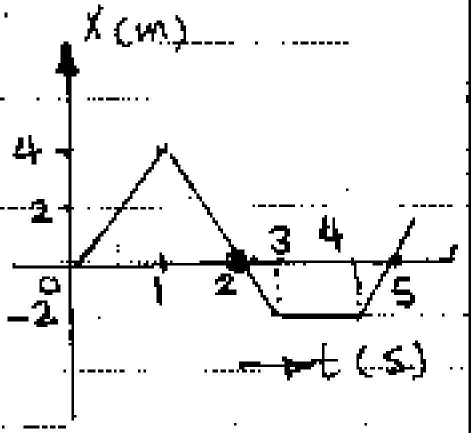
---

ex.4 /A

A tennis player moves in a straight line path as shown. Find the average velocity in the time intervals from

ⓐ 0 to 1 s ⓑ 0 to 4 s,

ⓒ 1 to 5 s and ⓓ 0 to 5 s



Ⓑ Find the instantaneous acceleration and velocities at ⓐ 5 s ⓑ 2 s ⓒ 3 s and ⓓ 4.5 s when $x = 3t^3 - 4t^2 + 8$

Sol Ⓐ

ⓐ $\bar{v} = \frac{\Delta x}{\Delta t} = \frac{4-0}{1-0} = 4\ m/s$ ⓑ $\bar{v} = \frac{-2-0}{4-0} = -0.5\ m/s$

ⓒ $\bar{v} = \frac{0-4}{5-1} = -1\ m/s$ ⓓ $\bar{v} = \frac{0-0}{5-0} = 0\ m/s$

Ⓑ $x = 3t^3 - 4t^2 + 8 \Rightarrow v = \frac{dx}{dt} = 9t^2 + 8t$

$a = \frac{dv}{dt} = 18t + 8$

Time(t), s	v (m/s)	a m/s²
2	20	28
3	27	46
4.5	146.25	73
5	185	82

ex5

A particle is moving along x-axis, its position varying with time as $x = 2t^3 - 3t^2 + t + 4$

ⓐ At what time is its velocity zero?

ⓑ what is the displacement at $t = 1s$, $t = 3s$?

ⓒ what is the velocity, acceleration at (2s) and the average speed from ($t = 1s$) to ($t = 3s$).

Solu

$x = 2t^3 - 3t^2 + t + 4$, $v = 6t^2 - 6t + 1$, $a = 12t - 6$

ⓐ $0 = 6t^2 - 6t + 1 \Rightarrow, t =$

ⓑ $x_1 = 2(1)^3 - 3(1)^2 + 1 + 4 \Rightarrow x_1 = 4m$

$x_2 = 2(3)^3 - 3(3)^2 + 1 + 4 \Rightarrow x_2 = 34m$

ⓒ

$V = 6(2)^2 - 6(2) + 1 \Rightarrow V = 13 m/s$

$a = 12(2) - 6 \Rightarrow a = 18 m/s^2$

At $t = 1s$, $x = 4m$, $t = 2s$, $x = 10m$

Average speed $= \frac{distance}{time} = \frac{4 + 10 + 34}{2} = 24 m/s$

Ex. 6

A police man moving on a highway with a speed of (30 Km/h) fires a bullet at thief's car speeding away in the same direction with a speed of (192 Km/h). If the muzzle speed of the bullet is (150 m/s) with what speed does the bullet hit the thief's car?

Solu

Relative velocity of thief's car with respect to the police car $= 192 - 30 = 162$ Km/h $= 45 m/s$

Muzzle speed of the bullet $= 150 m/s$

Relative velocity of the bullet with respect to the theif's car $= 180 - 45 = 105 m/s$

ex.7

Two towns (A) and (B) are (100 km) apart. A bus travels from (A) to (B) at (40 km/h) and returns from (B) to (A) at (50 km/h). calculate the average speed and average velocity of the bus.

Solu

Time taken to go from (A) to (B) $= \frac{100}{40} = 2.5\,h$

" " " " " (B) " (A) $= 100/50 = 2\,h$

$$\text{Average speed} = \frac{100+100}{2.5+2} = 44.4\ \text{Km/h}$$

$$\text{Average velocity} = \frac{100-100}{2.5+2} = 0\ \text{Km/h}$$

ex.8

A car starts from rest and travels for (5) s with acceleration of (1.5 m/s2). The driver then applies brakes for (3 s) causing acceleration of (−2 m/s2). (a) How fast of the car at the end of braking? how far has the car gone?

Solu $v = v_0 + at = 0 + (1.5\,m/s^2)(5\,s) = 7.5\,m/s$

(a) After braking $v_f = v_i + at = 7.5 + (-2\,m/s^2)(3\,s)$

$$v_f = 1.5\ m/s$$

(b) The total distance traveled is:-

$$\Delta X_{total} = \Delta X_1 + \Delta X_2 = \left(\frac{v+v_0}{2}\right)t_1 + \left(\frac{v_f+v_0}{2}\right)t_2$$

$$\Delta X_t = \left(\frac{7.5+0}{2}\right)(5) + \left(\frac{1.5+7.5}{2}\right)(3) = 32\,m$$

ex.9

A truck covers (40 m) in (8.50 s) while smoothly slowing down to a final velocity of (2.8 m/s). (a) Find the truck's original speed, (b) find its acceleration.

Solu

(a) $\Delta x = \bar{V}\Delta t = \left(\frac{V+V_0}{2}\right)\Delta t = \left(\frac{2.80+V_0}{2}\right)(8.50) = 40\,m$

$V_0 = 6.61\,m/s$

(b) $a = \frac{V-V_0}{\Delta t} = \frac{(2.80-6.61)}{(8.50)} = -0.448\,m/s^2$

ex.10

A motorist drives north for (35) minutes at (85 km/h) and then stops for (15) minutes. He then continues north, traveling (130 km) in (2) hours. (a) What is his total displacement? (b) What is his average velocity?

Solu

(a) Displacement $= \Delta x = V\Delta t = (85\,km/h)(35\,min)\left(\frac{1h}{60\,min}\right) + 130\,km$

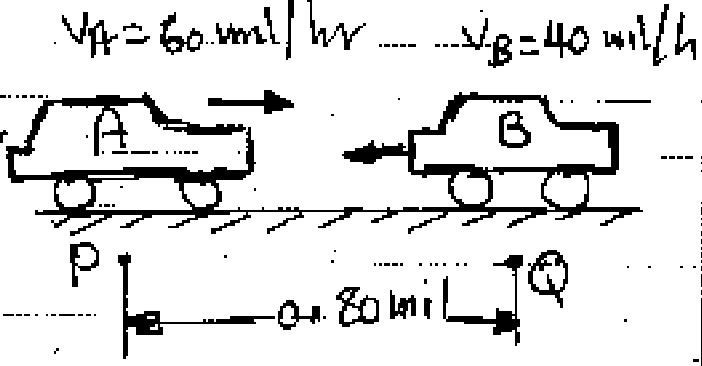
$\Delta x = 180\,Km$

(b) The total time $= (35\,min + 15\,min)\left(\frac{1h}{60\,min}\right) + 2\,hr$
$= 2.833\,hr$

$\vec{V} = \frac{\Delta x}{\Delta t} = \frac{180\,km}{2.833\,hr} = 63.53\,Km/hr$

ex.11

Two automobiles (A) and (B) are approaching each other in adjacent highway lanes.



At (t=0), (A) and (B) are (0.80) apart, their speeds as shown. Knowing that (A) passes point (Q), (50 s) after (B) was there, and that (B) passes point (P), (60 s) after (A) was there, determine: (a) the uniform acceleration of each car, (b) when the vehicles passes each other, and (c) the speed of each at that time.

Soln

(a) Car (A)

$$V_{Ai} = \frac{60*1609}{3600} = 26.817 \ m/s$$

$$X_f = 0.80*1609 = 1287.2 \ m$$

$$X_f = X_i + V_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow 1287.2 = 0 + 26.817(50) + \frac{1}{2}a(50)^2$$

$$a = -0.04292 \ m/s^2$$ deacceleration

Car (B)

$$V_{Bi} = \frac{40*1609}{3600} = 17.87 \ m/s$$

$$1287.2 = 0 + 17.87(60) + \frac{1}{2}(a)(60)^2$$

$$a = 0.119 \ m/s^2$$

(b) car (A) $X_f = 0 + 26.817t - \frac{1}{2} * 0.0429 (t^2)$

$$X_f = 26.817t - 0.02145t^2 \quad --(1)$$

Car (B)

$$(1287.2 - X_f) = 0 + 17.87t + \frac{1}{2} * 0.119t^2 \quad --(2)$$

equ (1) + eq (2)

$$0.03805t^2 + 44.687t - 1287.2 = 0$$

$t = 28.13$ sec ✓ $t = -1202.55$ ✗

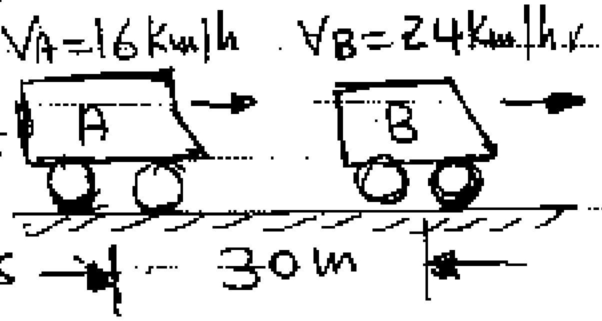
(c) $(V_f)_A = V_{Ai} + a_A t = 26.817 + (-0.0429)(28.13)$

$= 25.61$ m/s

$(V_f)_B = V_{Bi} + a_B t = 17.87 + (0.119)(28.13)$

$= 21.217$ m/s

Ex. 12



Two automobile (A) and (B) are traveling in adjacent high way lanes and at (t=0) have the positions and speeds as shown. knowing that automobile (A) has a constant acceleration of (0.8 m/s2) and that (B) has a constant deceleration of (0.5 m/s2), determine (a) when and where (A) will overtake (B), (b) the speed of each at that time.

Solu

(a) Car (A) $V_i = (16\ Km/hr) * \frac{1000}{3600} \Rightarrow V_i = 4.44\ m/s$

$X_f = 0 + 4.44t + \frac{1}{2}(0.8)t^2 \Rightarrow X_f = 4.44t + 0.4t^2$ --- (1)

$V_{xf} = V_{xi} + a_x t \Rightarrow V_{xf} = 4.44 + 0.8t$ --- (2)

Car (B)

$V_i = 24\ Km/h = 24/3.6 = 6.67\ m/s$

$X_f = 6.67t - 0.25t^2 + 30$ --- (3)

$V_{xf} = 6.67 - 0.5t$ --- (4)

equ (1) = equ (3)

$4.44t + 0.4t^2 = 30 + 6.67t - 0.25t^2$

$0.65t^2 - 2.23t - 30 = 0$

$t = -5.28\ sec$ X $t = 8.715\ sec$

substitute ($t = 8.715$ sec) in equ (1) or (3)

$X_f = 4.44(8.715) + 0.4(8.715)^2$

$X_f = 69\ m$

(b) speed of car (A)

$V_{xf} = 4.44 + (0.8)(8.715) \Rightarrow (V_{xf})_A = 41.08\ km/h$

Car (B)

$V_{xf} = 6.67 - (0.5)(8.715) = 2.3125\ m/s * 3.6$

$(V_{xf})_B = 8.325\ Km/hr$

ex. 13

A car moves in a straight line, such that for a short time its velocity is defined by $V = 0.2(6t^2 + 4t)$. Determine its position and acceleration when $t = 0$, $t = 5$ sec.

Solu

$V = 1.2t^2 + 0.8t = dx/dt$

$dx = (1.2t^2 + 0.8t)dt$

$\int dx = \int 1.2t^2\,dt + 0.8t\,dt$

$x = 0.4t^3 + 0.4t^2 + C$, at $t=0$, $x=0$, $C=0$

At $t = 5$ sec $\Rightarrow x = 0.4(5)^3 + 0.4(5)^2 \Rightarrow x = 60\,m$

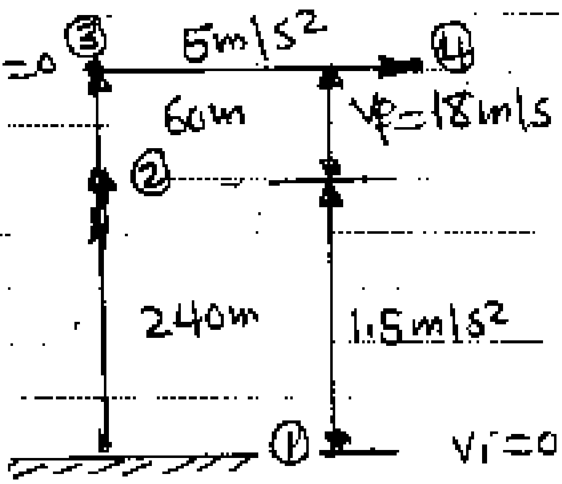
$a = dV/dt$

$a = 2.4t + 0.8$ at $t=0$, $a = 0.8\,m/s^2$

At $t = 5$ sec, $a = 2.4(5) + 0.8 \Rightarrow a = 12.8\,m/s^2$

Ex.14

A helicopter accelerates upward at ($1.5\,m/s^2$) to a height of (240 m). By time it reaches (300 m), it has accelerated to zero vertical velocity. It then accelerates horizontally at ($5\,m/s^2$) to a velocity of ($18\,m/s^2$). Calculate the total time of this trip.



Solu

Stage (1)

$x_f = x_i + V_{xi}t + \frac{1}{2}at^2 \Rightarrow 240 = 0 + 0 + \frac{1}{2}(1.5t^2)$.

$t = 17.88$ sec $= t_1$

At point (2) $V_{xf} = V_{xi} + at$

$V_{xf} = 0 + (1.5)(17.88) \Rightarrow V_{xf} = 19.38\,m/s$

Stage (2)

$v_{xf}^2 = u_{xi}^2 + 2a(x_f - x_i) \Rightarrow 0 = (19.38)^2 + 2a(300 - 240)$

$a = -3.129\ m/s^2$

$v_{xf} = u_{xi} + a_x t \Rightarrow 0 = 19.38 - 3.129t \Rightarrow t = 6.19\ s = t_2$

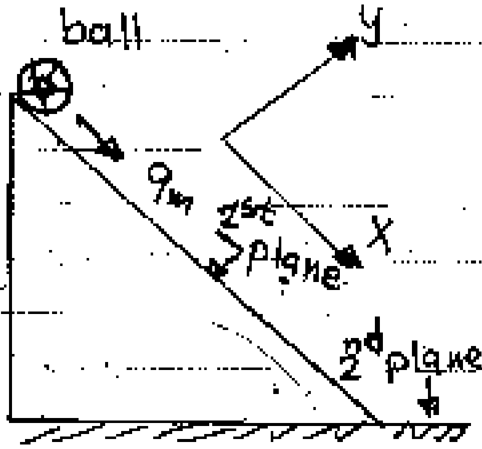
Stage (3)

$v_i = 0,\ v_f = 18\ m/s,\ a = 5\ m/s^2$

$v_{xf} = u_{xi} + a_x t \Rightarrow 18 = 0 + 5t \Rightarrow t = 3.6\ sec = t_3$

$t = t_1 + t_2 + t_3 = 17.88 + 6.19 + 3.6 = 27.67\ sec$

(Ex. 15)



A ball starts from rest and accelerates at (0.5 m/s²) while moving down on inclined plane (9 m) long. When it reaches the bottom, the ball rolls up another plane, where after moving (15 m) it comes to rest. (a) What is the speed of the ball at the bottom of the first plane, (b) how long does it take to roll down the first plane? (c) What is the acceleration along the second plane? (d) What is the ball speed (8 m) along the second plane?

(Sol.)

(a) For constant acceleration

$v_{xf}^2 = u_{xi}^2 + 2a_x(x_f - x_i),\ x_i = 0,\ x_f = 9m,\ u_{xi} = 0,\ a_x = 0.5\ m/s^2$

$v_{xf}^2 = 0 + 2(0.5)(9 - 0) \Rightarrow v_{xf} = 3\ m/s$

(b) For constant acceleration

$v_{xf} = u_{xi} + a_x t \Rightarrow 3\ m/s = 0 + (0.5\ m/s^2)t \Rightarrow t = 6\ sec$

ⓒ Along the second plane

$V_{xf}=0$, $U_{xi}=3$ m/s, $X_f=15$ m, $X_i=0$

$V_{xf}^2=U_{xi}^2+2a_x(X_f-X_i) \Rightarrow 0=(3)^2+2a_x(15-0)$

$a_x=-0.3$ m/s²

ⓓ Along the second plane

$x_i=0$, $X_f=8$ m, $V_{xi}=3$ m/s, $V_{xf}=?$ $a_x=-0.3$ m/s²

$V_{xf}^2=(3)^2+2(-0.3\text{ m/s}^2)(8-0)$

$V_{xf}=2.05$ m/s

(ex.16)

The acceleration of a particle is defined by the relation $(a=9-3t^2)$. The particle starts at $t=0$ with $V=0$ and $x=5$ m. Determine ⓐ the time when velocity is again zero, ⓑ the position and velocity when $(t=4\text{ sec})$, ⓒ the total distance traveled by the particle from $t=0 \rightarrow t=4$ s, ⓓ draw the x-t, v-t, a-t diagrams from $(t=0 \rightarrow t=4\text{ sec})$

(solu)

$a=9-3t^2=dv/dt \Rightarrow \int dv=\int 9\,dt-3t^2\,dt$

$v=9t-t^3+c_1$, at $t=0$, $V=0$, $\therefore c_1=0$

$V=9t-t^3=dx/dt \Rightarrow \int dx=\int 9t\,dt-t^3\,dt$

$X=4.5t^2-\frac{t^4}{4}+c_2$, at $t=0$, $X=5$ m, $c_2=5$

$X=4.5t^2-0.25t^4+5$

ts	Xm	V m/s	a m/s²	ts	Xm	V m/s	a
0	5	0	0	4	13	−28	−39
1	9.25	8	6	5	−38.75	−80	−66
2	19	10	−3				
3	25.25	0	−18				

ⓐ At $t=0$, $t=3$ sec, $V=0$ ⓑ At $t=4$, $X=13$ m

$V=-28$ m/s

ex.17
The motion of a particle is defined by the relation ($x = t^3 - 6t^2 + 9t + 5$), where x is expressed in feet and (t) in seconds. Determine (a) when the velocity is zero, (b) the position, acceleration and total distance traveled when ($t = 5s$), (c) total displacement when ($t = 5s$) (d) draw the (x-t), (v-t) and (a-t) curves.

Solu

$$x = t^3 - 6t^2 + 9t + 5 \quad ---- (1)$$
$$v = 3t^2 - 12t + 9 \quad --- (2)$$
$$a = 6t - 12 \quad --- (3)$$

(a) The time at which $v = 0$

$0 = 3t^2 - 12t + 9 \Rightarrow t = 1$ sec, $\Rightarrow t = 3$ sec

(b) $x = (5)^3 - 6(5)^2 + 9(5) + 5 \Rightarrow x = 25$ ft

$a = 6(5) - 12 \Rightarrow a = 18$ ft/s^2

(c) Total distance and total displacement

Initial position. at, $t = 0$, $x_0 = 5$ ft

final position at, $t = 5$ sec, $x_5 = 25$ ft

Displacement $\Delta x = x_f - x_i = 25 - 5 = 20$ ft

At $t = 1$ sec $x_1 = 9$ ft, at $t = 2$ sec $x_2 = 7$ ft

At $t = 3$ sec $x_3 = 5$ ft

At $t = 4$ sec $x_4 = 9$ ft

Total distance $= (x_1 + x_2 + x_3 + x_4 + x_5 - x_0) = (9 + 7 + 5 + 9 + 25 - 5)$

$= 50$ ft

t s	X ft	V ft/s	a ft/s²	t	X	V	a
0	5	9	−12	4	9	9	12
1	9	0	−6	5	25	24	18
2	7	−3	0	—	—	—	—
3	5	0	6	—	—	—	—

ex. 18

Which of the following cannot possibly be accelerating
(a) an object moving with a constant speed, (b) an object moving with a constant velocity, (c) an object moving along a curve.

Solu

(b) is correct, because ($\Delta V=0$) and $a=\frac{\Delta V}{\Delta t}=0$

ex. 19

Under which of the following conditions is the magnitude of the average velocity of a particle moving in one dimension smaller than the average speed over some time interval?

(a) A particle moves in the $+x$ direction without reversing.

(b) A particle moves in the $-x$ direction without reversing.

(c) A particle moves in the $+x$ direction and then reverses.

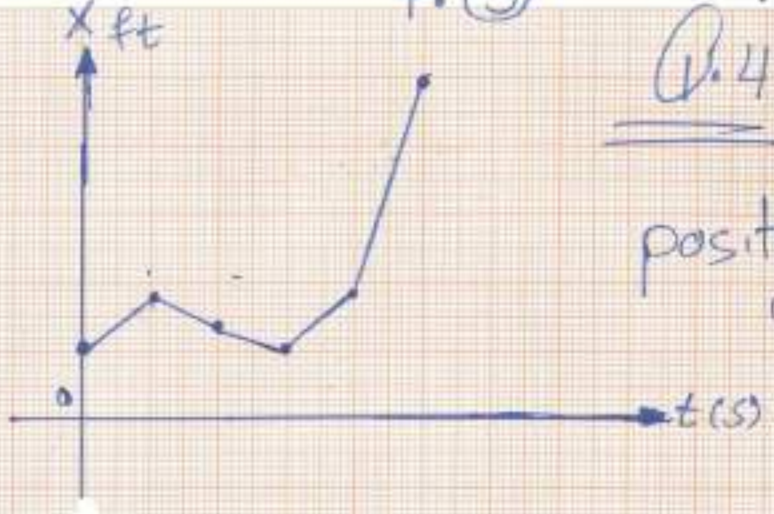
(d) There are no conditions for which this is true.

Solu

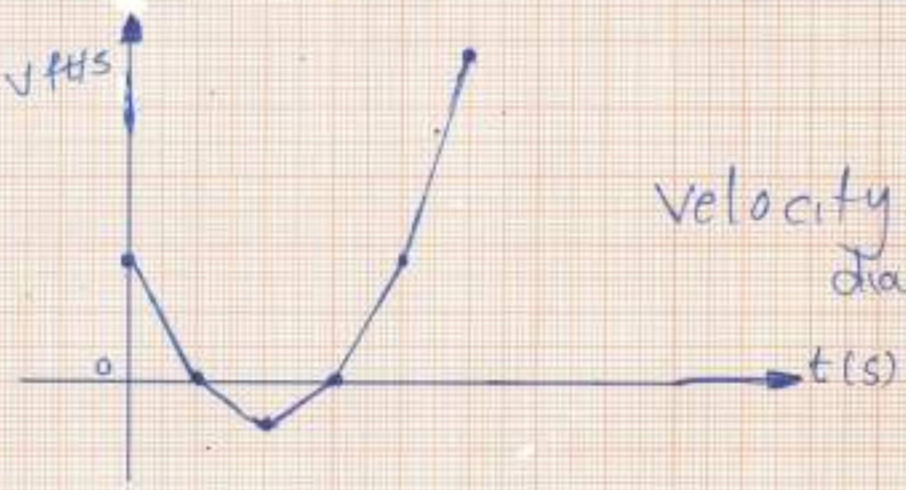
(c) is true. Displacement will be less than the distance travelled.

P. 5 44 29/2

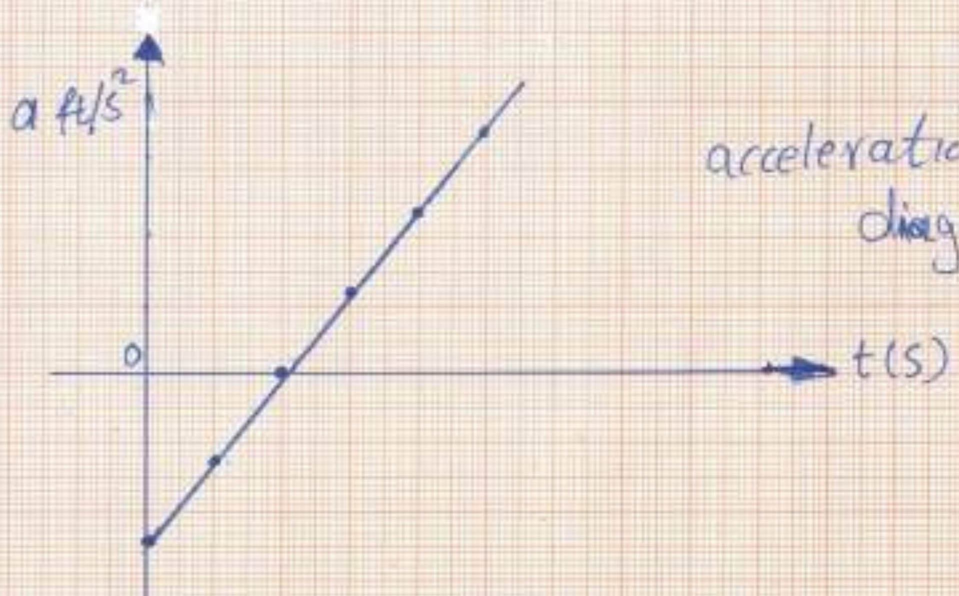
Q.4 civil Eng



position-time diagram



velocity-time diagram



acceleration-time diagram

Lecture

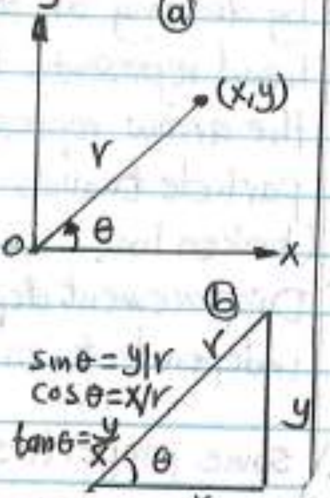
(4)

Chap. 3
Vectors

In this text we discuss the addition and subtraction of vector quantities together with some common applications to physical situations.

Coordinate systems

Cartesian coordinates (in which horizontal and vertical axes intersect at a point defined as the origin) are also called (rectangular coordinates). Sometimes it is more convenient to represent a point in a plane by its (plane coordinates r, θ) as shown in (fig 3.1/a). (r) is the distance from the origin to the point having cartesian coordinates, (x, y) and (θ) is the angle between a line drawn from the origin to the point and a fixed axis. This fixed axis is usually the positive (x axis) and (θ) is usually measured counterclockwise from it.



(Fig 3.1)

$x = r\cos\theta$, $y = r\sin\theta$, and $r = \sqrt{x^2+y^2}$

e.g

The cartesian coordinates of a point in the xy plane are $x = -3.50$ m, find the polar coordinates of this point. Take $y = -2.50$ m

Solu

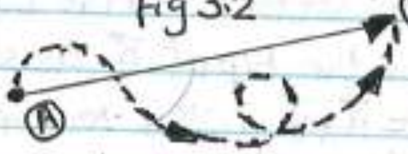
$r = \sqrt{x^2+y^2} = \sqrt{(-3.50)^2+(-2.50)^2} = 4.30$ m

$\tan\theta = y/x = -2.50\text{m}/-3.50\text{m} = 0.714 \Rightarrow \theta = \tan^{-1} 0.714 \Rightarrow \theta = 216^\circ$.

Vector and scalar quantities

A (scalar quantity) is completely specified by a single value with an appropriate unit and has no direction.

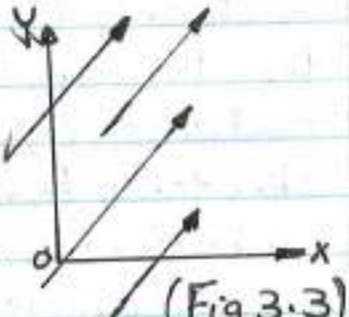
Fig 3.2



Examples of scalar quantities are (volume, speed, time, mass, temperature).

(vector quantity) is completely specified by a number and appropriate units plus a direction. Examples : (wind velocity, displacement). suppose a particle moves from some point Ⓐ to some point Ⓑ along a straight path as shown in (fig 3.2). we represent this displacement by drawing an arrow from Ⓐ to Ⓑ . The direction of the arrow head represents the direction of the displacement and the length of the arrow represents the magnitude of the displacement. If the particle travels along some other path from Ⓐ to Ⓑ such as the (broken line) its displacement is still the arrow drawn from Ⓐ to Ⓑ. Displacement depends only on the initial and final positions, it is independent on the path taken between these two points.

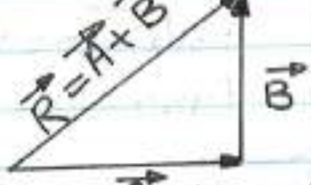
Some properties of vectors



(Fig 3.3)

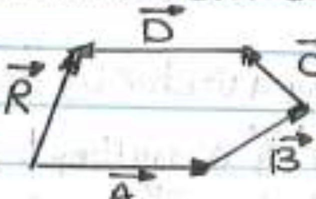
For many purposes, two vectors $\vec{A}$ and $\vec{B}$ may be defined to be equal if they have the same magnitude and point in the same direction. That is $\vec{A} = \vec{B}$ only if $A = B$ and $\vec{A}$ and $\vec{B}$ point in the same direction along parallel lines. All the vectors in (fig 3.3) are equal even though they have different starting points this property allow us to move a vector to a position parallel to itself in a diagram without affecting the vector.

Adding vectors (graphical method)



(Fig 3.4)

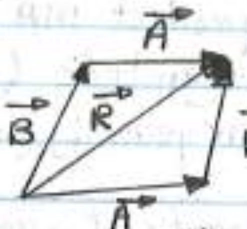
To add vector $\vec{B}$ to vector $\vec{A}$, first draw vector $\vec{A}$ on graph paper with its magnitude represented by a convenient length scale and then draw vector $\vec{B}$ to the same scale with its tail starting from the tip of $\vec{A}$, as shown in (Fig 3.4). The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is the vector drawn from the tail of $\vec{A}$ to the tip of $\vec{B}$. A geometric construction can also be used to add more than two vectors, as shown in (fig 3.5) for the case of four vectors. $\vec{R}$ is the vector drawn from the tail of the first vector to the tip of the last vector.



(fig 3.5)

when two vectors are added, the sum is independent of the order of the addition, the order is important when vectors are multiplied. This is known as the (commutative law of addition), fig 3.6

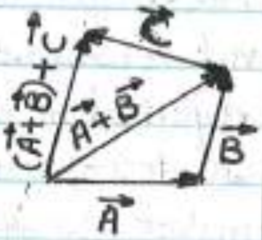
$\vec{A}+\vec{B}=\vec{B}+\vec{A}$, $\vec{R}=\vec{A}+\vec{B}=\vec{B}+\vec{A}$



(Fig 3.6)

when three or more vectors are added their sum is independent of the way in which the individual vectors are grouped together. This is called (associative law of addition), (Fig 3.7).

$\vec{A}+(\vec{B}+\vec{C})=(\vec{A}+\vec{B})+\vec{C}$



(Fig-7)

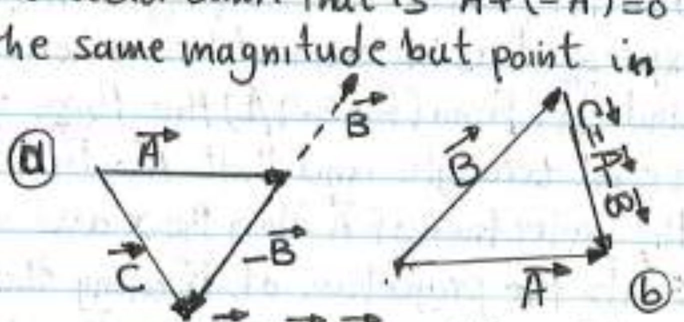
Negative of a vector

The negative of a vector $\vec{A}$ is defined as the vector that when added to $\vec{A}$ gives zero for the vector sum. That is $\vec{A}+(-\vec{A})=0$
The vectors $\vec{A}$ and $-\vec{A}$ have the same magnitude but point in opposite direction.

Subtracting vectors

We define the operation $\vec{A}-\vec{B}$ as vector $-\vec{B}$ added to vector $\vec{A}$

$$\vec{A}-\vec{B}=\vec{A}+(-\vec{B})$$



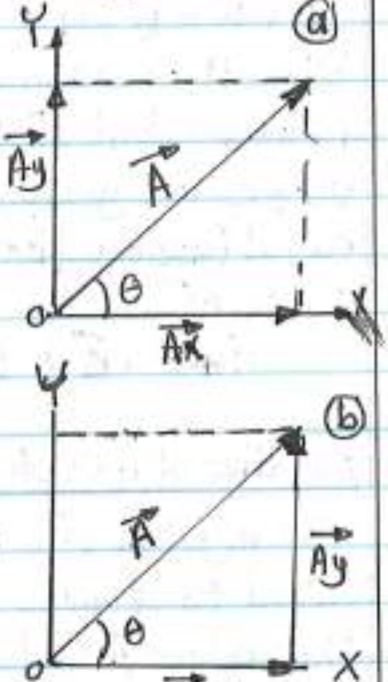
(Fig 3-8)

The geometric construction for substracting two vectors in this way is illustrated in (fig 3.8). A second way of looking at vector substraction is to note that the difference $\vec{A}-\vec{B}$ between two vectors $\vec{A}$ and $\vec{B}$ is add to the second vector to obtain the first (fig 3.8/b), the difference vector $\vec{C}$ points from the tip of the second vector to the tip of the first.

Multiplying a vector by a scalar

If vector $\vec{A}$ is multiplied by a positive scalar quantity (m), then the product $m\vec{A}$ is a vector that has the same direction as $\vec{A}$ and magnitude (mA). If vector $\vec{A}$ is multiply by a negative scalar quantity (-m), the the product $-m\vec{A}$ is directed opposite.

Components of a vector and unit vectors



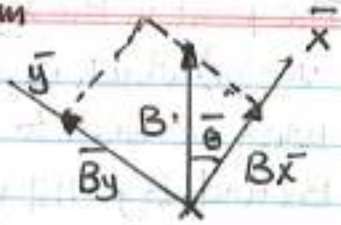
(Fig 3.9)

The graphical method of adding vectors is not recommonded whenever high accuracy is required or in three-dimensional problems. Here we describe a method of adding vectors that makes use of the projections (Components) of the vectors. Any vector can be completely described by its components. Vector $\vec{A}$ in the xy plane which making an arbitrary angle (θ) with the positive x-axis (fig 3.9/a) can be expressed as the sum of two other vectors $\vec{A}_x$ and $\vec{A}_y$. From (fig 3.9/b) the three vectors form a right triangle and that $\vec{A} = \vec{A}_x + \vec{A}_y$. Component $\vec{A}_x$ represents the projection of $\vec{A}$ along the x axis and the component $\vec{A}_y$ represents the projection of $\vec{A}$ along the y axis. These components can be positive or negative. From (fig 3.9) we see: $A_x = A\cos\theta$ and $A_y = A\sin\theta$. The signs of the components A_x and A_y depend on the angle (θ). for example if ($\theta = 120°$), then (A_x) is negative and (A_y) is positive. If ($\theta = 225°$) both (A_x) and (A_y) are negative.

$A = \sqrt{A_x^2 + A_y^2}$, $\theta = \tan^{-1} A_y/A_x$

Note: when we are working a physics problem that requires resolving a vector into its components, it is convenient to express the components in a coordinate system having axes that are not horizontal and vertical but are still perpendicular to each other. In (fig 3.10) vector $\vec{B}$ makes an angle ($\bar{\theta}$) with the $\bar{x}$ axis. The components of $\vec{B}$ along the $\bar{x}$ and $\bar{y}$ axes are:- $B_{\bar{x}} = B\cos\bar{\theta}$ and $B_{\bar{y}} = B\sin\bar{\theta}$, $B = \sqrt{B_{\bar{x}}^2 + B_{\bar{y}}^2}$

$\bar{\theta} = \tan^{-1}(B_{\bar{y}}/B_{\bar{x}})$.



(Fig 3-10)

Unit vectors

(Fig 3.11)

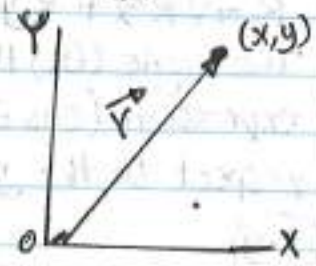
It is a dimensionless vector having a magnitude of exactly (1).

They are used to specify a given direction and have no other physical significance.

We shall use the symbols $\hat{i}, \hat{j}, \hat{k}$ to represent unit vectors pointing in the positive x, y, and z directions respectively.

(The "hats" on the symbols are a standard notation for unit vectors). The unit vectors $\hat{i}, \hat{j}$ and $\hat{k}$ forms a set of mutually perpendicular vectors in a right-handed coordinate system.

The magnitude of each unit vector equals (1), that is, $|\hat{i}| = |\hat{j}| = |\hat{k}| = 1$.



(Fig 3.12)

Consider a vector $\vec{A}$ lies in the xy plane as shown in (fig 3.11/b). The product of the component A_x and the unit vector $\hat{i}$ is the vector $A_x\hat{i}$ which lies on the x axis and has magnitude $|A_x|$. (The vector $\vec{A_x}\hat{i}$ is an alternative representation of vector $\vec{A_x}$). Thus the unit vector for the vector $\vec{A}$ is:

$$\vec{A} = \vec{A_x}\hat{i} + \vec{A_y}\hat{j}$$

The point in the xy plane (fig 3.12) can be specified by the (position vector) which is given by $\vec{r} = x\hat{i} + y\hat{j}$ $\vec{r}$ = position vector

when we wish to add vector $\vec{B}$ to $\vec{A}$, where vector $\vec{B}$ has components B_x and B_y. All we do is add the x and y components separately.

The resultant vector $\vec{R} = \vec{A} + \vec{B}$ is therefore :-

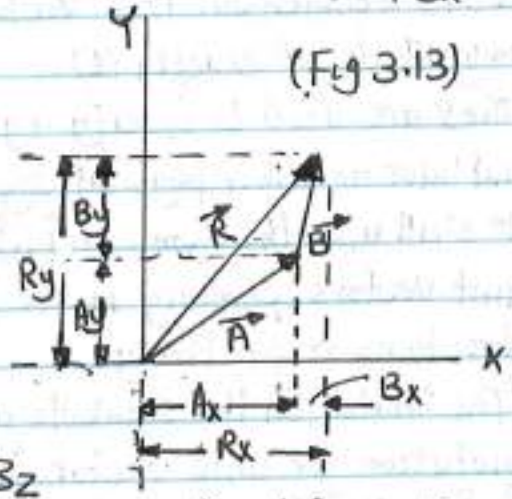
$\vec{R} = (A_x\hat{i} + A_y\hat{j}) + (B_x\hat{i} + B_y\hat{j})$ or

$\vec{R} = (A_x + B_x)\hat{i} + (A_y + B_y)\hat{j}$

Because $\vec{R} = \vec{R}_x\hat{i} + \vec{R}_y\hat{j}$ we see that the components of the resultant vector are:

$R_x = A_x + B_x$ and $R_y = A_y + B_y$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(A_x+B_x)^2 + (A_y+B_y)^2}$ and $\tan\theta = \frac{R_y}{R_x} = \frac{A_y + B_y}{A_x + B_x}$



(Fig 3.13)

when we need to consider situations involving motion in three component directions, we express them in the form: $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

The sum of $\vec{A}$ and $\vec{B}$ is

$R = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k}$

$R = \sqrt{R_x^2 + R_y^2 + R_z^2}$, $R_z = A_z + B_z$

The angle (θ_x) that (R) makes with x axis is found from the expression ($\cos\theta_x = \frac{R_x}{R}$) with similar expression for the angles with respect to the y and z axes.

e.g

Find the sum of two vectors $\vec{A}$ and $\vec{B}$ lying in the xy plane and given by :- $\vec{A} = (2.0\hat{i} + 2.0\hat{j})\,m$ and $\vec{B} = (2.0\hat{i} - 4.0\hat{j})\,m$

Solu) comparing with general expressions : $\vec{A} = A_x\hat{i} + A_y\hat{j}$ and $\vec{B} = B_x\hat{i} + B_y\hat{j}$ we see that $A_x = 2.0m$, $A_y = 2.0m$, $B_x = 2.0m$ and $B_y = -4.0m$

$\vec{R} = \vec{A} + \vec{B} = (2.0 + 2.0)\hat{i}\,m + (2.0 - 4.0)\hat{j}\,m$

$R = (4.0\hat{i} - 2.0\hat{j})\,m$ or $R_x = 4.0\,m$ and $R_y = -2.0\,m$

The magnitude of $\vec{R}$ is: $R = \sqrt{R_x^2 + R_y^2} = \sqrt{(4.0)^2 + (-2.0)^2} = \sqrt{20\,m^2}$

$R = 4.5\,m$

The direction of $\vec{R}$ is: $\tan\theta = \frac{R_y}{R_x} = \frac{-2.0\,m}{4.0\,m} = -0.50$

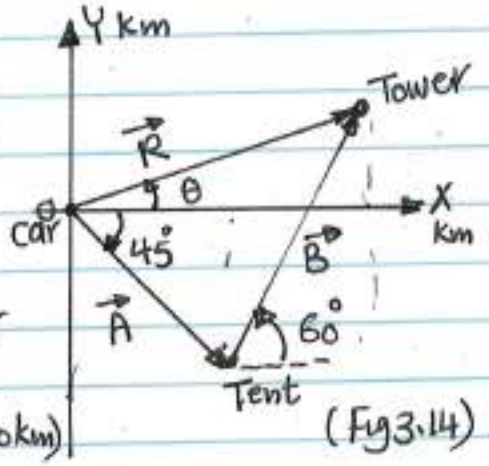
$\theta = \tan^{-1}(-0.5) \Rightarrow \theta = -27^\circ$ it means clockwise from the x axis.

Thus the angle for this vector is $(\theta = -27 + 360) \Rightarrow \theta = 333^\circ$ counter-clockwise from the x axis.

e.g

A hiker begins a trip by first walking (25.0 km) southeast from her car. she stops and sets up her tent for the night. On the second day, she walks (40.0 km) in a direction (60.0°) north of east, at which point she discovers a forest ranger's tower. (a) determine the components of the hiker's displacement for each day.

Solu



(Fig 3.14)

we denote the displacement vectors on the first and second days by $\vec{A}$ and $\vec{B}$ respectively, and use the car as the origin of coordinates.

We will analyse this problem by using our knowledge of vector components.

Displacement $\vec{A}$ has a magnitude of (25.0 km) and is directed (45.0°) below the positive x axis

$A_x = A\cos(-45.0^\circ) = (25.0\,km)(0.707) = 17.7\,km$.

$A_y = A\sin(-45.0^\circ) = (25.0\,km)(-0.707) = -17.7\,km$

The negative sign of A_y indicates that the hiker walks in the negative y direction on the first day. The second day displacement $\vec{B}$ has a magnitude of (40.0 km) and is (60.0°) north of east.

Its components are:

$B_x = B\cos\theta = (40.0\,km)(0.500) = 20.0\,km$,

$B_y = B\sin\theta = (40.0\,km)(0.866) = 34.6\,km$.

(b) Determine the components of the hiker's resultant displacement $\vec{R}$ for the trip. Find the expression of $\vec{R}$ in terms of unit vectors.

Solu

$\vec{R} = \vec{A} + \vec{B}$

$R_x = A_x + B_x = 17.7\ km + 20.0\ km = 37.7\ Km$

$R_y = A_y + B_y = -17.7\ km + 34.6\ Km = 16.9\ km$

In unit-vector form:-

$\vec{R} = (37.7\hat{i} + 16.9\hat{j})\ Km$

$R = \sqrt{R_x^2 + R_y^2} = \sqrt{(37.7)^2 + (16.9)^2} \Rightarrow R = 41.3\ Km$

$\tan\theta = \frac{R_y}{R_x} = \frac{16.9\ Km}{37.7\ Km} \Rightarrow \theta = 24.1°$ north of east.

Examples

ex. 1

Choose the correct response to make the sentence true :- a component of a vector is (a) always (b) never (c) sometimes larger than magnitude of the vector.

Solu.

(b) is correct. From the pythagoreum theorem the magnitude of a vector is always larger than the absolute value of each component, unless there is only one is non zero component, in which case the magnitude of the vector is equal to the absolute value of that component.

ex. 2

Consider the two vectors $\vec{A} = 5\hat{i} - 3\hat{j}$ and $\vec{B} = -2\hat{i} - 4\hat{j}$ Calculate (a) $\vec{A} + \vec{B}$, (b) $\vec{A} - \vec{B}$, (c) $|\vec{A} + \vec{B}|$, (d) $|\vec{A} - \vec{B}|$ (e) the direction of $|\vec{A} + \vec{B}|$ and (f) the magnitude and direction of $\vec{A} \cdot \vec{B}$

Solu

$$\vec{A}_x = 5\hat{i}, \quad \vec{B}_x = -2\hat{i}, \quad \vec{A}_y = -3\hat{j}, \quad \vec{B}_y = -4\hat{j}$$

(a) $\vec{A} + \vec{B} = (5-2)\hat{i} + (-3-4)\hat{j} \Rightarrow \vec{A} + \vec{B} = 3\hat{i} - 7\hat{j}$

(b) $\vec{A} - \vec{B} = (5+2)\hat{i} - 3\hat{j} - (-4)\hat{j} \Rightarrow \vec{A} - \vec{B} = 7\hat{i} + \hat{j}$

(c) $|\vec{A} + \vec{B}| = \sqrt{(Ax + Bx)^2 + (Ay + By)^2} = \sqrt{(3)^2 + (-7)^2} = 7.61$

(d) $|\vec{A} - \vec{B}| = \sqrt{(7)^2 + (1)^2} = 7.07$

(e) $\tan\theta = Ry/Rx = -7/3 \Rightarrow \theta = -66.8^\circ$

(f) $\vec{A}\cdot\vec{B} = AB\cos\theta$

$\vec{A}\cdot\vec{B} = (5\hat{i}-3\hat{j})\cdot(-2\hat{i}-4\hat{j}) = -10+12 = +2$

$A = \sqrt{A_x^2+A_y^2} = \sqrt{(5)^2+(-3)^2} = \sqrt{34}$

$B = \sqrt{B_x^2+B_y^2} = \sqrt{(-2)^2+(-4)^2} = \sqrt{20}$

$\theta = \cos^{-1}\dfrac{\vec{A}\cdot\vec{B}}{AB} = \dfrac{2}{\sqrt{20\times 34}} \Rightarrow \theta = 85.60$

Ex.3

Consider the three component vectors $\vec{A} = (3\hat{i}-3\hat{j})$m, $\vec{B} = (\hat{i}-4\hat{j})$m and $\vec{C} = (-2\hat{i}+5\hat{j})$m. calculate (a) the magnitude of $\vec{D} = \vec{A}+\vec{B}+\vec{C}$ (b) the magnitude of $\vec{E} = -\vec{A}-\vec{B}+\vec{C}$

Solu

$\vec{A}_x = 3\hat{i}$, $\vec{C}_x = -2\hat{i}$, $\vec{B}_x = \hat{i}$

$\vec{A}_y = -3\hat{j}$, $\vec{B}_y = -4\hat{j}$, $\vec{C}_y = +5\hat{j}$

(a) $\vec{D}_x = 3\hat{i}+\hat{i}-2\hat{i} = 2\hat{i}$

$\vec{D}_y = -3\hat{j}-4\hat{j}+5\hat{j} = -2\hat{j}$, $\vec{D} = +2\hat{i}-2\hat{j}$

$D = \sqrt{D_x^2+D_y^2} = \sqrt{(2)^2+(-2)^2} = \sqrt{8} = 2.82$

$\tan\theta = D_y/D_x = -2/2 = -1$ $\therefore \theta = \tan^{-1}(-1) = 45°$

(b) $\vec{E}_x = -3\hat{i}-\hat{i}-2\hat{i} = -6\hat{i}$, $\vec{E}_y = -(-3\hat{j})-(-4\hat{j})+5\hat{j} = 12\hat{j}$

$\vec{E} = -6\hat{i}+12\hat{j}$

$E = \sqrt{(-6)^2+(12)^2} = \sqrt{180} = 13.41$

$\tan\theta = E_y/E_x \Rightarrow \tan\theta = \frac{12}{-6}$, $\theta = \tan^{-1}(-2)$

$\theta = -63.43$

(ex. 4)
If at least one component of a vector is a positive number, the vector cannot (a) have any component that is negative (b) be zero (c) have three dimensions.

(Solu)
(b) is correct. From the pythagorean theory we see that the magnitude of a vector is non zero if at least one component is non zero.

(ex. 5)
The magnitude of two vectors be (4) and (5) and the value of their product is (10). Find the angle between them.

(Solu)

$|\vec{A}| = 4$, $|\vec{B}| = 5$

$\vec{A} \cdot \vec{B} = AB\cos\theta$

$$\cos\theta = \frac{\vec{A}\cdot\vec{B}}{AB} = \frac{10}{4\times 5} = 0.5 \Rightarrow \theta = \cos^{-1} 0.5 \Rightarrow \theta = 60°$$

(ex. 6)
A point is located in a polar coordinate system by ($r = 2.5m$, $\theta = 35°$). Find the (x,y) coordinates of the point.

(Solu)

$x = r\cos\theta = (2.5m)\cos 35° = 2.047m$

$y = r\sin\theta = (2.5m)\sin 35° = 1.433\ m$

(ex. 7) say yes or no

(a) The force is scalar quantity while the mass is vector quantity.

Solu No

ⓑ The scalar product of any two vectors is a vector quantity.

Solu . No

---

ex.8
The x and y components of vector $\vec{A}$ are 4 and 6 m. The x and y components of vector $(\vec{A}+\vec{B})$ are 10 and 9 m.
calculate for the vector $\vec{B}$
ⓐ its x and y components ⓑ its length
ⓒ the angle it makes with x-axis.

Solu
$A_x = 4m$, $A_y = 6m$, $(A_x + B_x) = 10m$, $(A_y + B_y) = 9m$

ⓐ Now $B_x = 6m$, and $B_y = 3m$

ⓑ $B = \sqrt{B_x^2 + B_y^2} = \sqrt{6^2 + 3^2} = \sqrt{45}\ m$

ⓒ $\theta = \tan^{-1} B_y/B_x = \tan^{-1} 3/6 = \tan^{-1} 0.5 \Rightarrow \theta = 26.6$

---

ex.9
Find the angle between vectors

$$\vec{A} = 3\hat{i} - 2\hat{j} + 5\hat{k}, \quad \vec{B} = -4\hat{i} + 3\hat{j} + 6\hat{k}$$

Solu
$\vec{A} \cdot \vec{B} = AB\cos\theta$
$\vec{A} \cdot \vec{B} = -12 - 6 + 30 = +12$

$|\vec{A}| = \sqrt{(3)^2 + (-2)^2 + 5^2} = 6.16$, $|\vec{B}| = \sqrt{(-4)^2 + (3)^2 + (6)^2} = 7.81$

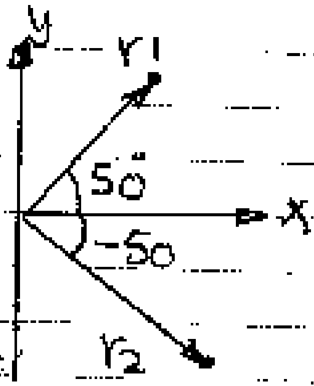
13/3 57

$\theta = \cos^{-1} \frac{12}{6.16 * 7.81} \Rightarrow \theta = \cos^{-1} 0.249 \Rightarrow \theta = 75.58$

ex. 10

Two points are given in polar coordinates by $[r_1, \theta_1 = (2, 50^\circ)]$ and $[r_2, \theta_2 = 5, -50^\circ]$ respectively. What is the distance between them? Use (r) is in meter.

Solu



$x_1 = r_1 \cos\theta_1 = 2 \cos 50 = 1.29\,m$

$y_1 = r_1 \sin\theta_1 = 2 \sin 50 = 1.53\,m$

$x_2 = r_2 \cos\theta_2 = 5 \cos(-50) = 3.21\,m$

$y_2 = r_2 \sin\theta_2 = 5 \sin(-50) = -3.83\,m$

$\Delta s = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(1.29 - 3.21)^2 + (1.53 + 3.83)^2}$

$= 5.69\,m$

ex. 11

Given $\vec{A} = 3\hat{i} - 4\hat{j} - 5\hat{k}$, $\vec{B} = \hat{i} + 2\hat{j} + \hat{k}$. find (a) $\vec{A} + \vec{B}$ (b) $\vec{A} - \vec{B}$ (c) $|\vec{A} + \vec{B}|$ and the direction.

Solu

(a) $\vec{A} + \vec{B} = (3\hat{i} + \hat{i}) + (-4\hat{j} + 2\hat{j}) + (-5\hat{k} + \hat{k})$

$= 4\hat{i} - 2\hat{j} - 4\hat{k}$

(b) $\vec{A} - \vec{B} = (3\hat{i} - \hat{i}) + (-4\hat{j} - 2\hat{j}) + (-5\hat{k} - \hat{k})$

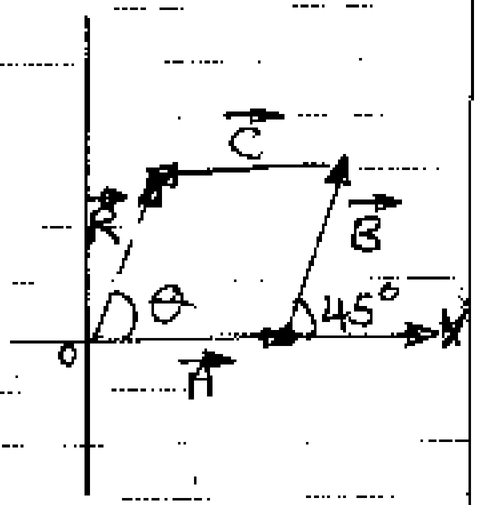
$= 2\hat{i} - 6\hat{j} - 6\hat{k}$

(c) $|\vec{A} + \vec{B}| = \sqrt{(4)^2 + (-2)^2 + (-4)^2} = \sqrt{36} = 6$

$\tan\theta = R_y / R_x = -2/4 = -0.5 \Rightarrow \theta = 26.56$ ↓x

ex.12

A man walks from a point (2 km) east then he walks (4 km) along a distance east north and finally he walks (3 km) toward west. Find the displacement from the starting point.



Solu

$Ax = 2\,Km$, $Ay = 0$

$Bx = B\cos 45 = 2.82\,Km$

$By = B\sin 45 = 2.82\,Km$

$Cx = -3\,Km$, $Cy = 0$

$Rx = 2 + 2.82 - 3 = 1.82\,km$

$Ry = 0 + 2.82 + 0 = 2.82\,Km$

$R = \sqrt{Rx^2 + Ry^2} = \sqrt{(1.82)^2 + (2.82)^2}$

$R = 3.35\,Km$

$\tan\theta = Ry/Rx \Rightarrow \theta = \tan^{-1}\frac{2.82}{1.82} \Rightarrow \theta = 57.16°$

ex.13

Find the polar coordinates corresponding to a point located at $(-5, 12)$m in cartesian coordinates.

Solu

$x = -5\,m$, $y = 12\,m$

$r = \sqrt{x^2 + y^2} = \sqrt{(-5)^2 + (12)^2} \Rightarrow r = 13\,m$

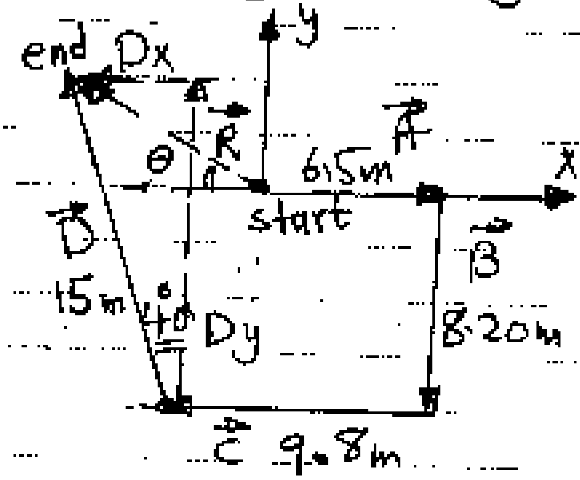
$\tan\theta = y/x = 12/-5 \Rightarrow \theta = -67.38$ $+x$

ex.14

A dog searching for a bone walks (6.5 m) east, then runs (8.20 m) south, follow by walks (9.8 m) west and then finally runs (15 m) at an angle

(40°) north of west. For this (2 min) trip, find (a) the total displacement vector, (b) the average speed and (c) the average velocity of the dog.

Solu (a)



$\vec{A}x = +6.5m$, $\vec{A}y = 0$
$\vec{B}x = 0$, $By = -8.2m$
$\vec{C}x = -9.8m$, $\vec{C}y = 0$
$\vec{D}x = -15 \sin 40 = -9.64m$
$\vec{D}y = 15 \cos 40' = +11.49m$

$\vec{R}x = Ax + Bx + Cx + Dx$
$= +6.5m + 0 - 9.8m - 9.6m = -12.94m$
$\vec{R}y = Ay + By + Cy + Dy = 0 - 8.2m + 0 + 11.49 = 3.29m$

$R = \sqrt{Rx^2 + Ry^2} = \sqrt{(-12.94)^2 + (3.29)^2} = 13.35m$

$\tan\theta = Ry/Rx = \frac{3.29m}{-12.94} \Rightarrow \theta = \tan^{-1}(-0.2542)$

$\theta = -14.26°$ or $\theta = (180 - 14.26) = 165.74$ from the east

(b)

$$\text{Average speed} = \frac{\text{total distance}}{\text{total time}} = \frac{6.5 + 8.20 + 9.8 + 15}{2 \times 60}$$
$$= \frac{39.5m}{120s} = 0.329 \text{ m/s}$$

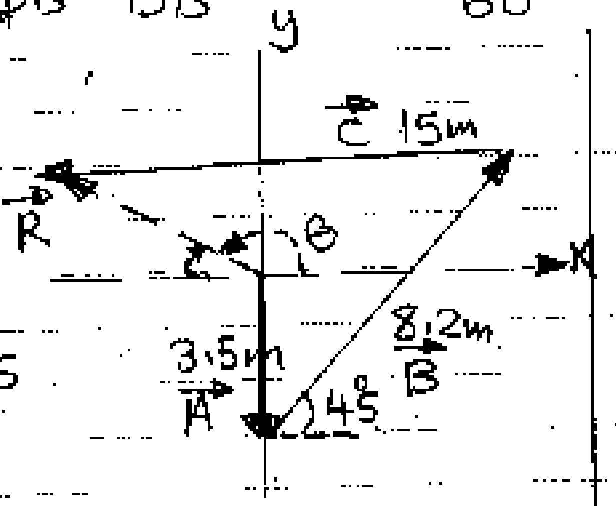
(c)

$$\text{Average velocity} = \frac{\text{Displacement}}{\text{total time}} = \frac{13.35m}{120 \text{ sec}}$$
$$= 0.111 \text{ m/s}$$

Ex.15

A particle undergoes the following displacements, (3.5m) south, (8.2m) north east and (15m) west.

What is the resultant displacement?



(solu)

$\vec{A_x} = 0$, $\vec{A_y} = -3.5m$

$\vec{B_x} = \vec{B}\cos 45 = 8.2\cos 45 = 5.798m$

$\vec{B_y} = \vec{B}\sin 45 = 8.2\sin 45 = 5.798m$

$\vec{C_x} = -15m$, $\vec{C_y} = 0$

$\vec{R_x} = \vec{A_x} + \vec{B_x} + \vec{C_x} \Rightarrow R_x = 0 + 5.798 - 15 = -9.202m$

$\vec{R_y} = \vec{A_y} + \vec{B_y} + \vec{C_y} = -3.5m + 5.798 + 0 = 2.298m$

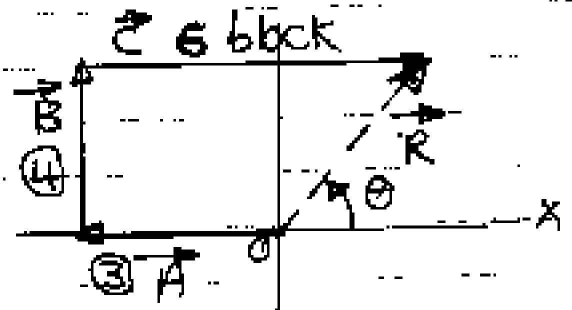
$R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(-9.202)^2 + (2.298)^2} = 9.484\ m$

$\theta = \tan^{-1} R_y / R_x \Rightarrow \theta = \tan^{-1}\frac{2.298}{-9.202} \Rightarrow \theta = -14^{\circ}$

(or) $\theta = -14^{\circ} + 180 = 166^{\circ}$ +x

(ex.16)

A girl delivering newspapers covers her rout by traveling (3) block west, (4) block north and then (6) blocks east. (a) what is her resultant displacement? (b) what is the total distance she travels?



(Solu) (a)

$\vec{A_x} = -3$, $\vec{A_y} = 0$

$\vec{B_x} = 0$, $\vec{B_y} = +4$

$\vec{C_x} = 6$, $\vec{C_y} = 0$

$\vec{R_x} = \vec{A_x} + \vec{B_x} + \vec{C_x} = -3 + 0 + 6 = 3$ blocks

$\vec{R_y} = \vec{A_y} + \vec{B_y} + \vec{C_y} = 0 + 4 + 0 = 4$ block

$\vec{R} = \vec{R_x} + \vec{R_y} = 3\hat{i} + 4\hat{j}$

$R = \sqrt{Rx^2 + Ry^2} = \sqrt{(3)^2 + (4)^2} \Rightarrow R = 5$ block

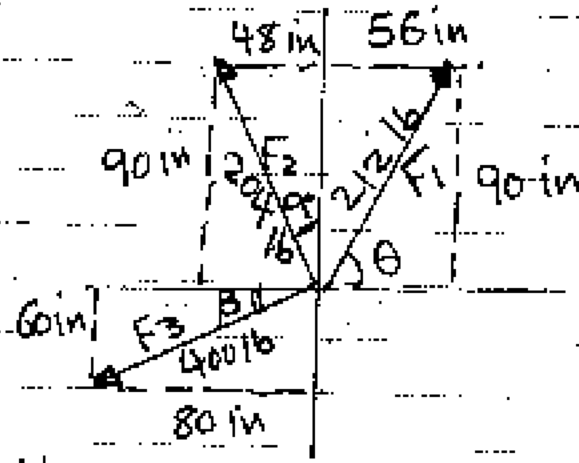
(b) $\theta = \tan^{-1} Ry / Rx \Rightarrow \theta = \tan^{-1} 4/3 \Rightarrow \theta = 53.129^\circ$ north of east

Distance $= Ax + By + Cx = 3 + 4 + 6 = 13$ block

(ex. 17)

Determine the magnitude and direction of the resultant of the forces acting as shown.

(Solu)



$\tan\theta = \frac{90}{56} = 1.607$

$\theta = \tan^{-1} 1.607 \Rightarrow \theta = 58.1$

$(F_1)x = F_1 \cos 58.1$
$= 112$ lb

$(F_1)y = F_1 \sin 58.1$
$= 180$ lb

or $(F_1)y = \sqrt{F_1^2 - F_x^2} = 180$ lb

$\tan\alpha = \frac{48}{90} = 0.53 \Rightarrow \alpha = 28.07^\circ$

$(F_2)x = F_2 \sin 28.07 \Rightarrow (F_2)x = 96$ lb

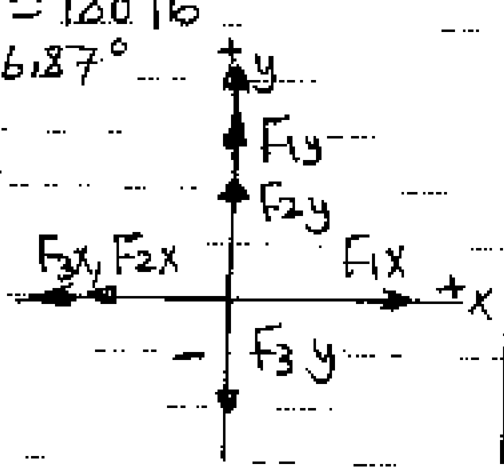
$(F_2)y = F_2 \cos 28.07 \Rightarrow (F_2)y = 180$ lb

$\tan\beta = 60/80 = 0.75 \Rightarrow \beta = 36.87^\circ$

$(F_3)x = F_3 \cos 36.87$
$= 320$ lb

$(F_3)y = F_3 \sin 36.87$
$= 240$ lb

$Rx = \Sigma Fx = F_1x + F_2x + F_3x$
$= (112 - 96 - 320)$
$= -304$ lb



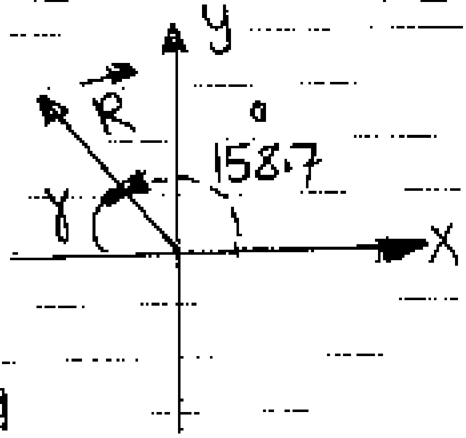
$R_y = \Sigma F_y = F_{1y} + F_{2y} + F_{3y}$
$= 180 + 180 - 240$
$= 120$ lb

$R = \sqrt{R_x^2 + R_y^2}$

$R = \sqrt{(-304)^2 + (120)^2}$

$R = 326.82$ lb

$\tan\gamma = R_y / R_x = \frac{120}{-304} = -0.39$



$\gamma = \tan^{-1}(-0.39) \Rightarrow \gamma = -21.30°$

or $(180 - 21.30) = 158.7°$ relative to the $+X$ positive X-axis counter clockwise.

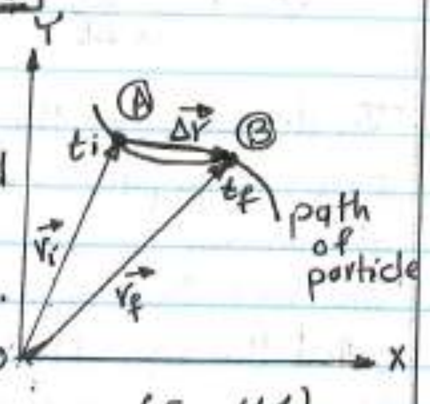
Lecture

(5)

Chap. 4
Motion In TWO Dimensions

The position, velocity and acceleration vectors

The position of the particle is described by its (position vector) (r), drawn from the origin of some coordinate system to the particle located in the xy plane. At time (ti) the particle is at point Ⓐ described by position vector ($\vec{r_i}$). At some later time (t_f) it is at point Ⓑ described by position vector ($\vec{r_f}$). The path from Ⓐ to Ⓑ is not necessarily a straight line, As the particle moves from Ⓐ to Ⓑ in the time interval ($\Delta t = t_f - t_i$) its position vector changes from ($\vec{r_i}$) to ($\vec{r_f}$).



(Fig 4.1)

The displacement of the particle is (displacement vector, $\vec{\Delta r}$), It is the difference between its final and its initial position vectors.

$$\vec{\Delta r} = \vec{r_f} - \vec{r_i}$$

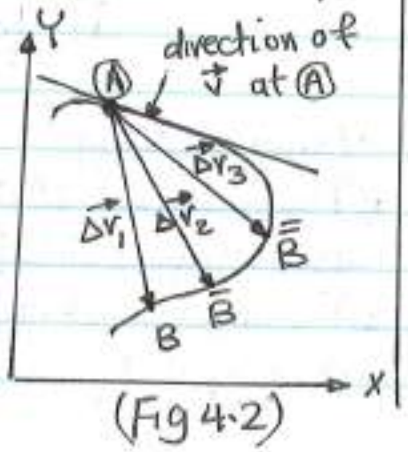
The magnitude of ($\vec{\Delta r}$) is less than the distance traveled along the curved path followed by the particle.

Average velocity is $\quad \vec{\bar{v}} = \frac{\vec{\Delta r}}{\Delta t}$

$\bar{v}$ = average velocity that is a vector quantity along $\vec{\Delta r}$.

<u>Note</u>, the average velocity between points is independent of the path taken, because, It is proportional to displacement which depends only on the initial and final position vectors.

As a particle moves between two points, Its average velocity is in the direction of the displacement vector $\vec{\Delta r}$ which become smaller and smaller as the end point of the path is moved from Ⓑ to $\bar{B}$ to $\bar{\bar{B}}$, and Δt approach Zero when the end point approach Ⓐ, (Fig 4.2).



(Fig 4.2)

The direction of the displacement approaches that of the line tangent to the path at (A). The instantaneous velocity $\vec{v}$ is defined as (the limit of the average velocity $\Delta\vec{r}/\Delta t$ as Δt approach zero).

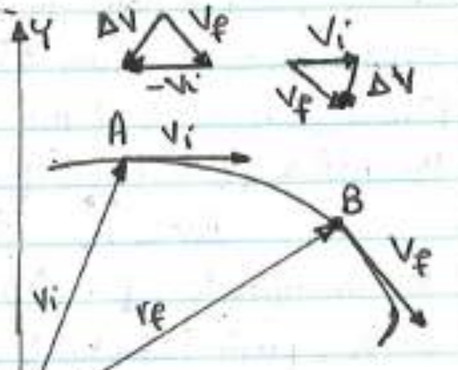
$$\vec{v} = \lim_{\Delta t \to 0} \frac{\Delta \vec{r}}{\Delta t} \quad \text{or} \quad v = \frac{dr}{dt}$$

The direction of the instantaneous velocity vector at any point in a particle's path is along a line tangent to the path in the direction of motion.

The magnitude of the instantaneous velocity vector $v = |\vec{v}|$ is called the (speed) which is a scalar quantity.

The average acceleration ($\vec{a}$) of a particle is defined as (the change in the instantaneous velocity vector $\Delta\vec{v}$ divided by the time interval during which that change occurs.

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta \vec{v}}{\Delta t}$$



(Fig 4.3)

The average acceleration is a vector quantity directed along $\Delta\vec{v}$.

The direction of $\Delta\vec{v}$ is found by adding the vector $-\vec{v}_i$ (the negative of $\vec{v}_i$) to the vector $\vec{v}_f$ because by defination

$$\Delta\vec{v} = \vec{v}_f - \vec{v}_i$$

The instantaneous acceleration is defined as the (limiting value of the ratio $\Delta\vec{v}/\Delta t$ as Δt approach zero).

$$\vec{a} = \lim_{\Delta t \to 0} \frac{\Delta \vec{v}}{\Delta t} = \frac{dv}{dt}$$

Various changes can occur when a particle accelerates :-

1) The magnitude of the velocity vector (speed) may change with time as in straight-line (one-dimensional motion).
2) The direction of the velocity vector may change with time even if its magnitude (speed) remains constant, as in curved path

(two-dimensional) motion.

3) Both the magnitude and the direction of the velocity vector may change simultaneously.

Two-Dimensional Motion With constant Acceleration

The position vector for a particle moving in the xy plane can be written

$$\vec{r} = x\hat{i} + y\hat{j}$$

where x, y and r change with time as the particle moves while the unit vectors $\hat{i}$ and $\hat{j}$ remains constant. If the position vector is known, the velocity of the particle is:

$$\vec{v} = \frac{dr}{dt} = \frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j} = v_x\hat{i} + v_y\hat{j}$$

Because $(\vec{a})$ is assumed constant, its components (a_x) and (a_y) also are constants. for constant acceleration substitute $(v_{xf} = v_{xi} + a_x t)$ and $(v_{yf} = v_{yi} + a_y t)$ in the above equation we obtain the final velocity at any time (t) is :-

$$\vec{v_f} = (v_{xi} + a_x t)\hat{i} + (v_{yi} + a_y t)\hat{j} = (v_{xi}\hat{i} + v_{yi}\hat{j}) + (a_x\hat{i} + a_y\hat{j})t$$

$\vec{v_f} = \vec{v_i} + \vec{a}t$ equ. of velocity vector as a function of time.

The results states that the velocity of a particle at some time (t) equals the vector sum of its initial velocity $\vec{v_i}$ and the additional velocity $(\vec{a}t)$ at time (t) as a result of constant acceleration.

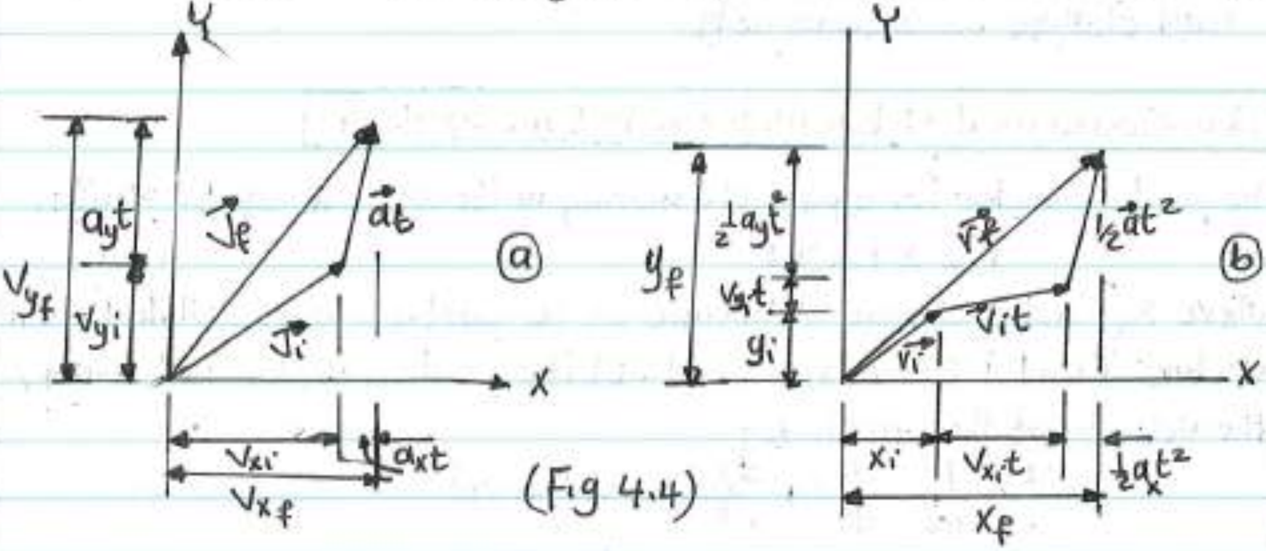
Similarly by using $x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$ and $y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$ substitute in the equ. $r_f = x_f + y_f$, we obtain :-

$\boxed{\vec{r_f} = \vec{r_i} + \vec{v_i}t + \frac{1}{2}at^2} \rightarrow$ equ. of position vector as funtion of time.

where $r_i = (x_i\hat{i} + y_i\hat{j})$, $v_i = (v_{xi}\hat{i} + v_{yi}\hat{j})$ and $a = (a_x\hat{i} + a_y\hat{j})$

$\vec{r_f}$ = position vector, $\vec{r_i}$ = original position, $v_i t$ = displacement arising from the initial velocity of the particle and displacement $(\frac{1}{2}\vec{a}t^2)$ resulting from the constant acceleration of the particle.

we see from (fig 4.4/a) that $\vec{v_f}$ is generally not along the direction of either $\vec{v_i}$ or $\vec{a}$ because the relationship between these quantities is a vector expression. Also $\vec{r}_f$ is generally not along the direction of $\vec{v}_i$ or $\vec{a}$. ($\vec{v}_f$ and $\vec{r}_f$) are generally not in the same direction.



(Fig 4.4)

The components of the equations for $\vec{v}_f$ and $\vec{r}_f$ show us that two-dimensional motion at constant acceleration is equivalent to two independent motions, one in the x direction and one in the y direction, having constant acceleration a_x and a_y.

e.g.

A particle starts from the origin at ($t=0$) with an initial velocity having an (x) component of (20 m/s) and a (y) component of (−15 m/s). The particle moves in the xy plane with an x component of acceleration only by ($a_x = 4.0\,m/s^2$).

ⓐ Determine the components of the velocity vector at any time and the total velocity vector at any time.

Solu) The particle moving in two dimensions with constant acceleration. The particle starts by moving toward the right and downward.

$V_{xi} = 20\,m/s$, $V_{yi} = -15\,m/s$, $a_x = 4\,m/s^2$, $a_y = 0$

$V_{xf} = V_{xi} + a_x t = (20 + 4.0\,t)\,m/s$

$V_{yf} = V_{yi} + a_y t = -15\,m/s + 0 = -15\,m/s$

$V_f = V_{xf}\hat{i} + V_{yf}\hat{j} = [(20 + 4.0t)\hat{i} - 15\hat{j}]\,m/s$

(Fig 4.5)

(b) Calculate the velocity and speed of the particle at ($t=5.0\,s$).

(Solu)

$V_f = [(20+4.0(5.0)\hat{i} - 15\hat{j}]\,m/s = (40\hat{i} - 15\hat{j})\,m/s$

or at $t=5$ second, $V_{xf} = 40\,m/s$ and $V_{yf} = -15\,m/s$

To determine the angle (θ) that $\vec{V}$ makes with the x axis at ($t=5.0\,s$) is: $\tan\theta = V_{yf}/V_{xf} \Rightarrow \theta = \tan^{-1}\frac{V_{yf}}{V_{xf}} = \tan^{-1}\left(\frac{-15\,m/s}{40\,m/s}\right) = -21^\circ$.

The speed is the magnitude of $\vec{V_f}$

$V_f = |\vec{V_f}| = \sqrt{V_{xf}^2 + V_{yf}^2} = \sqrt{(40)^2+(-15)^2} = 43\,m/s.$

(c) Determine the x and y coordinates of the particle at any time (t) and the position at this time.

(Solu) Because $x_i = y_i = 0$ at $t=0$

$\therefore x_f = V_{xi}t + \frac{1}{2}a_x t^2 = (20t + 2t^2)\,m$ and $y_f = V_{yi}t = (-15t)\,m$

$r_f = x_f\hat{i} + y_f\hat{j} = [(20t+2t^2)\hat{i} - 15t\hat{j}]\,m$

At $t=5.0\,s$, $x = 150\,m$, $y = -75\,m$ and $\vec{r_f} = (150\hat{i} - 75\hat{j})\,m$

The magnitude of the displacement of the particle from the origin at $t=5.0\,s$ is the magnitude of $\vec{r_f}$ at this time:-

$r_f = |\vec{r_f}| = \sqrt{(150)^2+(-75)^2} = 170\,m.$

Projectile Motion

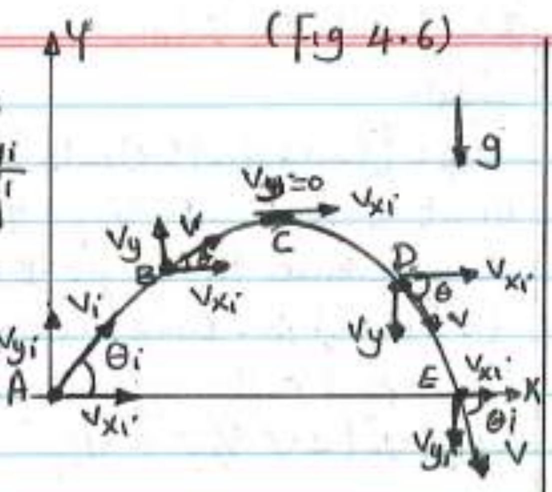
In this text we use two assumptions: 1) the free-fall acceleration (g) is constant over the range of motion [that is small compared with the radius of the Earth ($6.4\times10^6\,m$)] and is directed downward, 2) the effect of air resistance is negligible.

With these assumptions, we find that the path of a projectile which we call its (trajector) is always a (parabola).

To show that the trajectory of a projectile is a parabola, let us choose the y direction is vertical and positive is upward ($a_y = -g$) as in (one-dimensional free fall) and that ($a_x = 0$). Let us assume that at ($t=0$) the projectile leaves the origin ($x_i = y_i = 0$) with speed

(V_i) as shown in (fig 4.6).

(Fig 4.6)



The vector $\vec{v_i}$ makes an angle θ_i with horizontal, $\cos\theta_i = V_{xi}/V_i$ and $\sin\theta_i = \frac{V_{yi}}{V_i}$
substituting $x_i = 0$ and $a_x = 0$, we find that $x_f = v_{xi}t \Rightarrow \boxed{x_f = (V_i\cos\theta_i)t}$
And by using $y_i = 0$ and $a_y = -g$
we obtain:- $y_f = v_{yi}t + \frac{1}{2}a_y t^2$

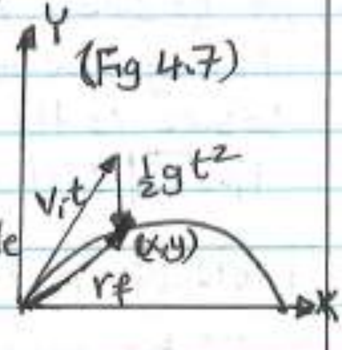
$$\boxed{y_f = (V_i\sin\theta_i)t - \frac{1}{2}gt^2}$$

when substitute $t = x_f / V_i\cos\theta_i$ we obtain

$$\boxed{y = (\tan\theta_i)x - \left(\frac{g}{2V_i^2\cos^2\theta_i}\right)x^2}$$

This equation is valid for launch angles in the range $0 < \theta < \pi/2$, and this equation is of the form $(y = ax - bx^2)$ which is the equation of a parabola that passes throug the origin.

(Fig 4.7)



The position vector of the projectile as a function of time is:-

$$\vec{r_f} = \vec{r_i} + \vec{v_i}t + \frac{1}{2}gt^2$$

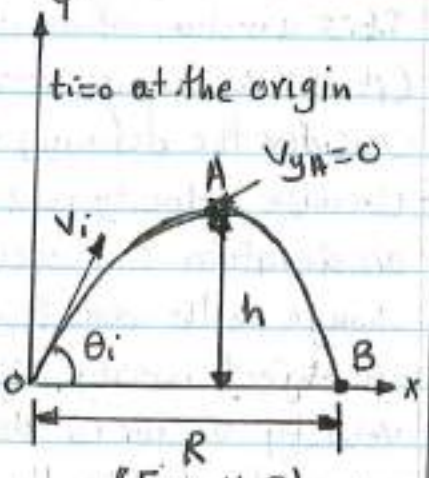
If there is no gravitational acceleration, the particle would continue to move along a straight path in the direction of $\vec{v_i}$.

projectile motion is a special case of two dimensional motion with constant acceleration $(a_y = -g)$ in the y direction and with zero acceleration in the x direction. It is considered to be the superposition of two motions:-
1) constant-velocity motion in the horizontal direction, and 2) free-fall motion in the vertical direction.

The horizontal and vertical components of a projectile's motion are completely independent of each other.

Horizontal Range and Maximum Height of a Projectile

v_i = initial velocity, h = maximum height of the projectile, R = horizontal range. At (A) the peak of the trajectory, the particle has coordinates $(R/2, h)$.
A projectile is launched from the origin with a positive (v_{yi}) component (fig 4.8).
The time (t_A) to which the projectile reaches the peak is :- $v_{yf} = v_{yi} + a_y t \Rightarrow 0 = v_i \sin\theta_i - g t_A$



(Fig 4.8)

$$\boxed{t_A = \frac{v_i \sin\theta_i}{g}}$$

The expression of (h) in terms of the magnitude and direction of the initial velocity vector is :- $y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$
Take: $y_f = y_A = h$, $y_i = 0$ and $t = t_A = v_i \sin\theta_i / g$, $a_y = -g$

$$h = (v_i \sin\theta_i)(v_i \sin\theta_i / g) - \tfrac{1}{2} g \left(\frac{v_i \sin\theta_i}{g}\right)^2$$

$$\boxed{h = v_i^2 \sin^2\theta_i / 2g}$$

The range (R) is the horizontal position at a time (t) that is (twice) the time at which it reaches its peak, that is at point (B), $t_B = 2t_A$
By substitutate : $v_{xi} = v_{xB} = v_i \cos\theta_i$, and $x_B = R$ at $t_B = 2t_A$ we find :-

$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$, $a_x = 0$, $x_i = 0$, $x_f = x_B = R$ $\therefore x_f = v_{xi} t$ or

$$R = v_{xB} t_B = (v_i \cos\theta_i) 2t_A \Rightarrow R = (v_i \cos\theta_i) \frac{2 v_i \sin\theta_i}{g} = \frac{2 v_i^2 \sin\theta_i \cos\theta_i}{g}$$

Using $\sin 2\theta = 2 \sin\theta \cos\theta$

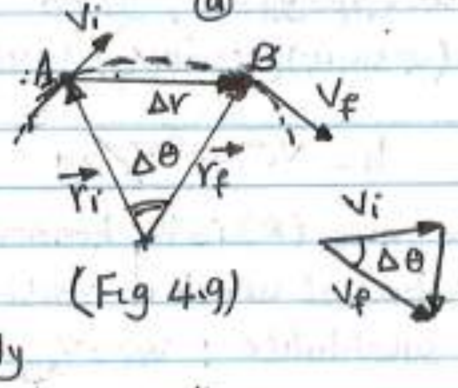
$$\Rightarrow \boxed{R = \frac{v_i^2 \sin 2\theta_i}{g}}$$

The maximum value of (R) is $\left(R_{max} = \frac{v_i^2}{g}\right)$ at $\sin 2\theta_i = 1$ or at $2\theta_i = 90^\circ$, $\theta_i = 45^\circ$

Uniform Circular Motion

It's a motion of an object in a circular path with constant speed. (it still has an acceleration).
Consider the defining equation for average acceleration, $\bar{a} = \frac{\Delta v}{\Delta t}$.
Because velocity is a vector quantity, there are two ways in which an acceleration can occur by a change in the magnitude and/or by a change in the direction of the velocity. The latter situation occurs for an object moving with constant speed in a circular path. The velocity vector is always tangent to the path of the object and perpendicular to the radius of the circular path. The acceleration vector in uniform circular motion is always perpendicular to the path and always points toward the center of the circle. This acceleration is called a (centripetal acceleration) and it's magnitude is

$$\boxed{a_c = v^2/r}$$



(Fig 4.9)

r = circle radius
consider a particle follows a circular path from Ⓐ to Ⓑ. At Ⓐ ($t = t_i$), velocity = v_i. At Ⓑ $t = t_f$, $v = v_f$.
Assume that (v_i) and (v_f) differ only in direction ($v = v_i = v_f$), their magnitudes are the same.
To calculate the acceleration of the particle we have :-

$$\vec{a} = \frac{v_f - v_i}{t_f - t_i} = \frac{\Delta v}{\Delta t}$$

The velocity vector ($\vec{v}$) is always perpendicular to the position vector ($\vec{r}$). The two triangles in (fig 4.9 a+b) are similar because the angle between any two sides is the same for both triangles and the ratio of the lengths of these sides is the same $\frac{|\Delta v|}{v} = \frac{|\Delta r|}{r}$.
where $v = v_i = v_f$ and $r = r_i = r_f$

The magnitude of the average acceleration over the time interval for the particle move from Ⓐ to Ⓑ is:

$$|\bar{a}| = \frac{|\Delta v|}{\Delta t} = \frac{v}{r}\frac{|\Delta r|}{\Delta t}$$

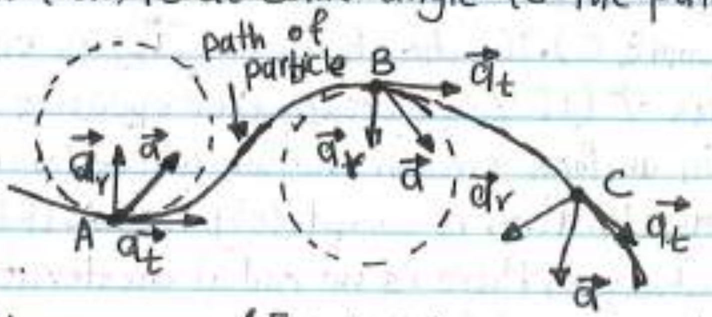
As Ⓐ and Ⓑ approach each other, (Δt) approach (zero) and the ratio $\boxed{\frac{|\Delta r|}{\Delta t}}$ approach the speed (v). The average acceleration becomes the instantaneous acceleration at point Ⓐ.
Hence in the limit $\Delta t \to 0$, the magnitude of the acceleration is:

$$a_c = \frac{v^2}{r}$$

The time required for one complete revolution is: $T = \frac{2\pi r}{v}$, this is because speed is equal to the circumference of the circular path divided by the period.

Tangent and Radial Acceleration

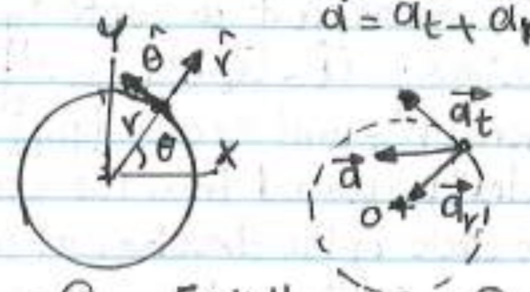
Consider a particle moving along a smooth curved path where the velocity changes both in direction and in magnitude (Fig 4.10).
In this situation the velocity vector is always tangent to the path, the acceleration vector ($\vec{a}$) is at some angle to the path.
At each of three points Ⓐ, Ⓑ, and Ⓒ, we draw dashed circles that represent a portion of the actual path at each point. The radius of the circles is equal to the radius of the curvature of the path at each point.



(Fig 4-10)

$\vec{a} = \vec{a}_t + \vec{a}_r$

As the particle moves along the curved path, the direction of the total acceleration vector ($\vec{a}$) changes from point to point.



ⓐ Fig 4-11 ⓑ

This vector can be resolved into components based on an origin at the center of the dashed circle : a radial component ($\vec{a}_r$) along the radius of the model circle and a tangential component ($\vec{a}_t$) perpendicular to this radius.

$$\vec{a} = \vec{a}_r + \vec{a}_t$$

($\vec{a}_t$) causes the change in the speed of the particle and it is parallel to the instantaneous velocity and is given by:-

$$a_t = \frac{d|V|}{dt}$$

($\vec{a}_r$) arises from the change in direction of the velocity vector and is given by:-

$$a_r = -a_c = \frac{-V^2}{r}$$

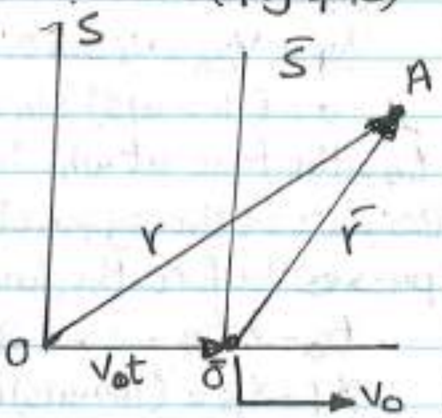
r = radius of curvature of the path at the point in question.
The direction of (a_c) is toward the center of the circle while the direction of radial unit vector ($\hat{r}$) is always points away from the center of the circle.
The magnitude of ($\vec{a}$) is $\boxed{a = \sqrt{a_r^2 + a_t^2}}$. At a given speed ($a_r$) is large when the radius of curvature is small at points Ⓐ and Ⓑ in (fig 4-10) and small when (r) is large (such as at point, C). The direction of ($\vec{a}_t$) is either in the same direction as $\vec{v}$ (if V is increasing) or opposite $\vec{v}$ (if V is decreasing).
In uniform circular motion where (V) is constant, $a_t = 0$ and the acceleration is completely radial. If the direction of ($\vec{v}$) does not change, there is no radial acceleration and the motion is one-dimensional (in this case $a_r = 0$, $a_t \neq 0$)
It is convenient to write the acceleration in terms of unit vectors: $\vec{a} = \vec{a}_t + \vec{a}_r = \frac{d|V|}{dt}\hat{\theta} - \frac{V^2}{r}\hat{r}$
where $\hat{r}$ = unit vector lying along the radius vector and directed radially outward from the center. $\hat{\theta}$ = unit vector tangent to the circle in the direction of increasing (θ). Both ($\hat{\theta}$) and ($\hat{r}$) move along with the particle and so vary in time.

Relative Velocity and Relative Acceleration

An example of this concept is the motion of a package dropped from an airplane flying with a constant velocity. An observer on the airplane sees the motion of the package as a straight line downward to ward Earth. The stranded explorer on the ground, however sees the trajectory of the package as a parabola. Once the package is dropped, and the airplane continues to move horizontally with the same velocity, the package hits the ground directly beneath the airplane. we assume that air resistance is neglected.

Consider a particle located at point (A) in (fig 4-12). Imagine that the motion of this particle is being described by two observers, one in reference frame (S), fixed relative to Earth, and another in reference frame ($\bar{S}$), moving to the right relative to (S) (and therefor relative to Earth) with a constant velocity v_o [relative to an observer in ($\bar{S}$), (S) moves to the left with a velocity $-v_o$]. we define ($t=0$) as that instant at which the origins of the two reference frames coincide in space. Thus at time (t), the origins of the reference frames will be separated by a distance ($v_o t$). we label the position of the particle relative to the (S) frame with the position vector ($\vec{r}$) and that relative to the ($\bar{S}$) frame with the position vector ($\vec{\bar{r}}$) both at time (t).

(Fig 4-12)



The two vectors are related to each other through the expression: $\vec{r} = \vec{\bar{r}} + \vec{v}_o t$ or $\vec{\bar{r}} = \vec{r} - \vec{v}_o t$

If we differentiate this equation relative to time and note that $\vec{v}_o$ is constant

We obtain: $\frac{d\bar{r}}{dt} = \frac{dr}{dt} - v_o \Rightarrow \boxed{\vec{\bar{v}} = \vec{v} - \vec{v}_o}$

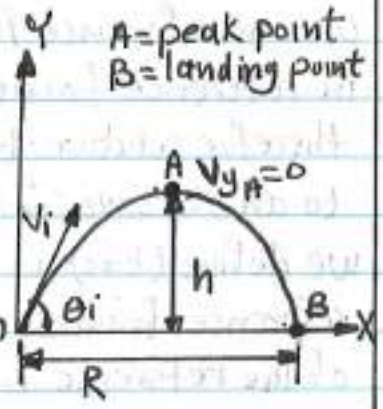
$\vec{\bar{v}}$ = velocity of the particle observed in the $(\bar{S})$ frame, $\vec{v}$ = velocity of the particle observed in the (S) frame. These two equations are known as (Galilean transformation equations). Although observers in two frames measure different velocities for the particle, they measure the same acceleration when $\vec{v}_o$ is constant.

$$\frac{d\bar{v}}{dt} = \frac{dv}{dt} - \frac{dv_o}{dt} \Rightarrow \frac{dv_o}{dt} = 0 \text{ because } v_o = \text{constant}$$

$$\text{and } \bar{a} = \frac{d\bar{v}}{dt},\ a = \frac{dv}{dt}$$

we conclude: $\vec{\bar{a}} = \vec{a}$ (the acceleration of the particle measured by an observer in one frame of reference is the same as that measured by any other observer moving with constant velocity relative to the first frame).

Examples



ex-1

A long-jumper leaves the ground at an angle of (20°) above the horizontal and at a speed of $11.0\,m/s$.

(a) How far does the jump in the horizontal direction?

(Solu) $x_f = x_B = (v_i \cos\theta_i)t_B = (11.0\,m/s)(\cos 20^\circ)t_B$

we can find (t_B) by remembering that $(a_y = -g)$ and by using $V_{y_A} = 0$ at the top of the jump

$V_{y_f} = V_{y_A} = v_i \sin\theta_i - g\,t_A$

$0 = (11.0\,m/s)\sin 20 - 9.80\,m/s^2\, t_A \Rightarrow t_A = 0.384\,s$

t_A = the time at which the long-jumper is at the top of the jump. Because of the symmetry of the vertical motion another $(0.384\,s)$ passes before the jumper return to the ground. Therefore:

$t_B = 2\,t_A = 2 * 0.384 = 0.768\,s$.

$x_f = x_B = (11.0\,m/s)(\cos 20)(0.768\,s) = 7.94\,m$

~~This is a reasonable answer (distance) for a world-class athlete~~

(b) what is the maximum height reached.

(solu) $y_{max} = y_A = (v_i \sin\theta_i)t_A - \frac{1}{2} g t_A^2$

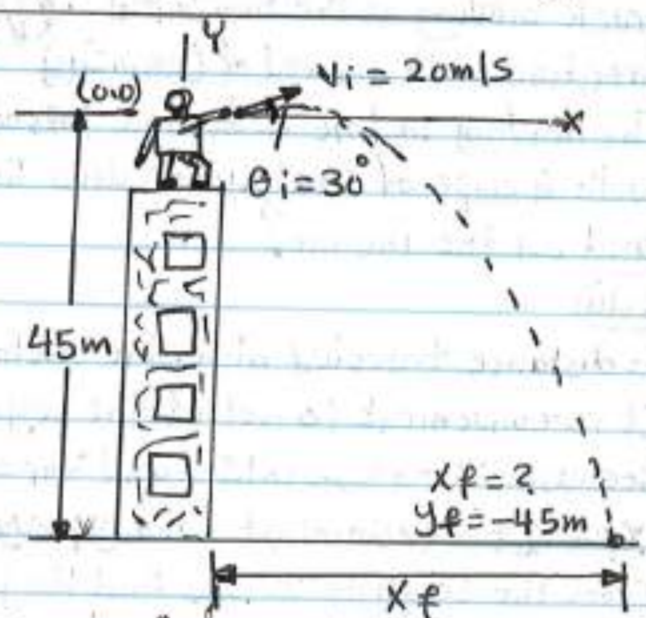
$= (11.0\,m/s)(\sin 20^\circ)(0.384\,s) - \frac{1}{2}(9.80\,m/s^2)(0.384)^2$

$= 0.722\,m$

Check the results by using: $h = \frac{v_i^2 \sin^2\theta_i}{2g}$ and $R = \frac{v_i^2 \sin 2\theta_i}{g}$

(e.x 2)

A stone is thrown from the top of a building upward at an angle of (30.0°) to the horizontal with an initial speed of (20.0 m/s) and the height of the building is (45.0 m).



(a) How long does it take the stone to reach the ground?

(solu)

$v_{xi} = v_i \cos\theta_i = (20.0\,m/s)(\cos 30.0^\circ) = 17.3\,m/s$

$v_{yi} = v_i \sin\theta_i = (20.0\,m/s)(\sin 30.0^\circ) = 10.0\,m/s$

To find (t) we use $y_f = y_i + v_{yi}t + \frac{1}{2} a_y t^2$ with $y_i = 0$, $y_f = -45\,m$, $a_y = -g$ and $v_{yi} = 10.0\,m/s$

$-45.0\,m = (10.0\,m/s)t - \frac{1}{2}(9.80\,m/s^2)t^2$

It is quadratic equation for (t). $\Rightarrow t = 4.22\,s$.

(b) What is the speed of the stone just before it strikes the ground?

(solu) we use the equation $v_{yf} = v_{yi} + a_y t$ with $t = 4.22\,s$ to obtain (y) component of the velocity just before the stone strikes the ground.

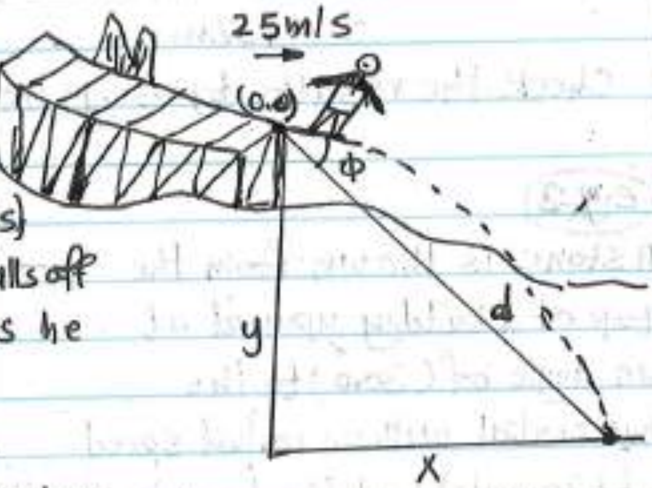
$v_{yf} = 10.0\,m/s - (9.80\,m/s^2)(4.22\,s) = -31.4\,m/s$

Because $v_{xf} = v_{xi} = 17.3\,m/s$, the required speed is

$v_f = \sqrt{v_{xf}^2 + v_{yf}^2} = \sqrt{(17.3)^2 + (-31.4)^2} = 35.9\ m/s$

It is reasonable answer because the final speed is greater than the initial speed of (20.0 m/s).

(ex-3)



A ski-jumper leaves the ski track moving in the horizontal direction with a speed of (25.0 m/s). The landing incline below him falls off with a slope of (35°). where does he land on the incline?

(solu)

d = distance traveled along the incline.

It is convenient to select the begining of the jump as the origin.

Because $v_{xi} = 25.0\ m/s$ and $v_{yi} = 0$, then :-

$x_f = v_{xi} t = (25.0\ m/s)t$ and $y_f = v_{yi} t + \frac{1}{2} a_y t^2 = -\frac{1}{2}(9.80\ m/s^2)t^2$

From the triangle we see that the jumpers x and y coordinates at the landing point are : $x_f = d\cos 35°$ and $y_f = -d\sin 35°$

$d\cos 35° = (25\ m/s)t$ and $-d\sin 35 = -\frac{1}{2}(9.80\ m/s^2)t^2$

$\therefore d = 109\ m,\ x_f = 89.3\ m,\ y_f = -62.5\ m$

(ex-4) Tangent and Radial Acceleration

A car exhibits a constant acceleration of (0.300 m/s²) parallel to the roadway. The car passes over a rise in the road way such that the top of the rise is shaped like a circle of radius (500 m). At the moment the car is at the top of the rise, its velocity vector is horizontal and has a magnitude of (6.00 m/s). what is the direction of the total accleration vector for the car at this instant?

(solu)

Because the car is moving along a curved path, we can categorize this as a problem involving a particle experiencing both tangential and radial acceleration.

$a_r = -\frac{v^2}{r} = -\frac{(6.00 m/s)^2}{500 m} = -0.0720 m/s^2$

The radial acceleration vector is directed downward while the tangential acceleration vector has magnitude of $(0.300 m/s^2)$ and is horizontal.

Because $\vec{a} = \vec{a}_r + \vec{a}_t$, the magnitude of $\vec{a}$ is :-

$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(-0.072)^2 + (0.300)^2}$

$a = 0.309 m/s^2$

If (ϕ) is the angle between $(\vec{a})$ and the horizontal, then

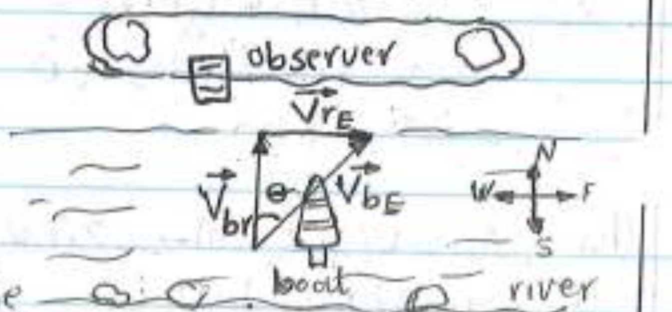
$\phi = \tan^{-1}\frac{a_r}{a_t} = \tan^{-1}\left(\frac{-0.072 m/s^2}{0.300 m/s^2}\right) \Rightarrow \phi = -13.5^\circ$.

(e.x-5) Relative velocity and Relative Acceleration

A boat heading due to north crosses a wide river with a speed of (10.0 Km/h) relative to the water. The water in the river has a uniform speed of (5.00 Km/h) due east relative to the Earth. Determine the velocity of the boat relative to an observer standing on either bank.

(solu)

Imaging moving across river while the current pushes you allong the river. You will not be able to move directly across the river but will end up down stream.



Because of the separate velocities of you and the river, we can categorize this as a problem involving relative velocities.

$\vec{V}_{br}$ = the velocity of the boat relative to the rive, $\vec{V}_{rE}$ = the velocity of the river relative to Earth, $\vec{V}_{bE}$ = the velocity of the boat relative to Earth.

$\vec{V}_{bE} = \vec{V}_{br} + \vec{V}_{rE}$

$V_{bE} = \sqrt{(V_{br})^2 + (V_{rE})^2} = \sqrt{(10.0)^2 + (5.00)^2} = 11.2 \text{ km/h}$

The direction of $\vec{V}_{bE}$ is $\theta = \tan^{-1}\left(\frac{V_{rE}}{V_{br}}\right) = \tan^{-1}\left(\frac{5.00}{10.00}\right) = 26.6°$

The boat is moving at a speed of (11.2 km/h) in the direction (26.6°) east of north relative to Earth.

Ex. 6

A ball is tossed from an upper-story window of a building. The ball is given an initial velocity of (8 m/s) at an angle of (20°) below the horizontal. It strikes the ground (3 s) later. (a) How far horizontally from the base of the building does the ball strike the ground? (b) Find the height from which the ball was thrown, (c) how long does it take the ball to reach a point (10 m) below the level of launching?

Take: $g = 9.80 \text{ m/s}^2$

Solu $v_i = 8$ m/s

(a) $v_{xi} = v_i \cos(-20) = 7.517$ m/s

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2$

$= 0 + (7.517)(3s) + 0$

$= 22.55$ m

(b) $v_{yi} = v_i \sin(-20) = -2.736$ m/s

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$a_y = -g$

$y_f = 0 - (2.736)(3) - \frac{1}{2}(9.80)(3)^2$

$y_f = -52.308$ m

(c) $y_f = -10$ m, $t = ?$

$-10 = 0 - (2.736)t - \frac{1}{2}(9.80)t^2 \Rightarrow 4.9t^2 + 2.736t - 10 = 0$

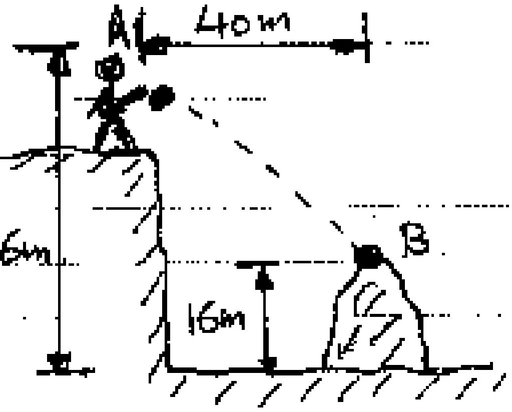
Lecture

(6)

$\therefore t = 1.18$ sec

Ex-7

With what minimum horizontal velocity (V_0) can a boy throw a rock at (A) and have it just clear the obstraction at (B)? Take $g = 9.80\ m/s^2$



Solu

Take the reference point at (A), so that:-

$x_f = 40m,\ x_i = 0,\ y_i = 0,\ y_f = -(26-16) = -10m$

$y_f = y_i + v_{yi} t + \frac{1}{2} a_y t^2$

$-10 = 0 + 0 - \frac{1}{2}(9.80) t^2 \Rightarrow t = 1.428$ sec

$x_f = x_i + v_{xi} t + \frac{1}{2} a_x t^2$

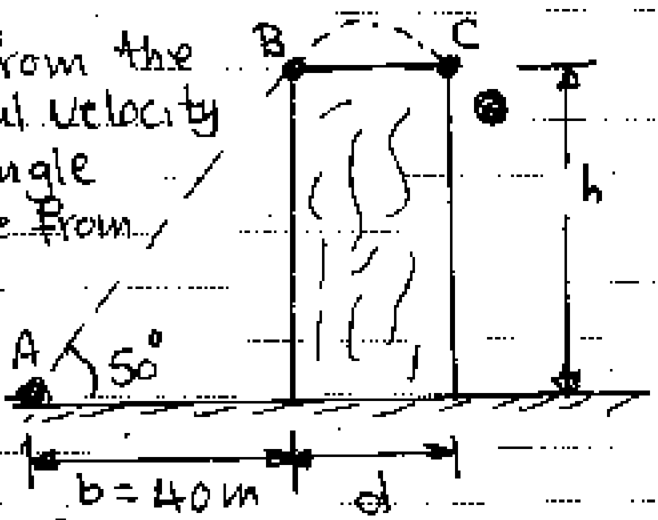
$40 = 0 + v_{xi}(1.428) + 0 \Rightarrow v_{xi} = v_0 \simeq 28\ m/s$

Ex-8

A ball is thrown upward from the level ground with an initial velocity of (30m/s) and thrown angle of ($\theta = 50°$). The distance from thrown point (A) to the building is ($b = 40m$).

Calculate:-

(a) at what height does the ball strike the building? (b) the width of the building.



Solu

$v_i = 30 m/s,\ v_{xi} = v_i \cos\theta = 30 \cos 50 \Rightarrow v_{xi} = 19.28\ m/s$

$v_{yi} = v_i \sin\theta = 30 \sin 50 \Rightarrow v_{yi} = 22.98\ m/s$

when $x_f = 40m$

$40 = 0 + (19.28)t + 0 \Rightarrow t = 2.07\ sec$

$y_f = 0 + (22.98)(2.07) - \frac{1}{2}(9.80)(2.07)^2$

$y_f = 26.55\ m$

when $y_f = 26.55\ m$

$26.55 = 0 + (22.98)t - \frac{1}{2}(9.80)t^2 \Rightarrow t = 2.069\ sec$ X

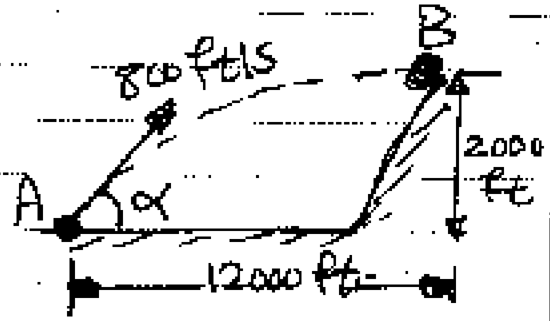
$t = 2.615\ sec$ ← use

$x_f = (40 + d) = 0 + (19.28)(2.615) + 0$

$d = 10.44\ m$

(ex.9)

A projectile is fired with an initial velocity of (800 ft/s) at a target (B) located (2000 ft) above the gun (A) and at a horizontal distance of (12000 ft). Neglecting air resistance, determine the value of the firing angle (α). Use $g = 32.2\ ft/s^2$



(Solu). Horizontal Motion

$V_{xf} = V_{x0} = V_i \cos\alpha = 800 \cos\alpha$

$x_f = x_i + v_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow 12000 = 0 + 800\cos\alpha\, t + 0 \Rightarrow$

$t = \frac{15}{\cos\alpha}$

Vertical Motion

$V_{yi} = V_{y0} = 800 \sin\alpha$

$y_f = y_i + V_{yi}t - \frac{1}{2}gt^2 \Rightarrow 2000 = 0 + (800 \sin\alpha)\left(\frac{15}{\cos\alpha}\right) - \frac{1}{2} \times (32.2)\left(\frac{15}{\cos\alpha}\right)^2$

Since $\frac{1}{\cos^2\alpha} = \sec^2\alpha = 1 + \tan^2\alpha$, we have:

$2000 = 800(15)\tan\alpha - 16.1(15)^2(1 + \tan^2\alpha)$

$3622\tan^2\alpha - 12000\tan\alpha + 5622 = 0$

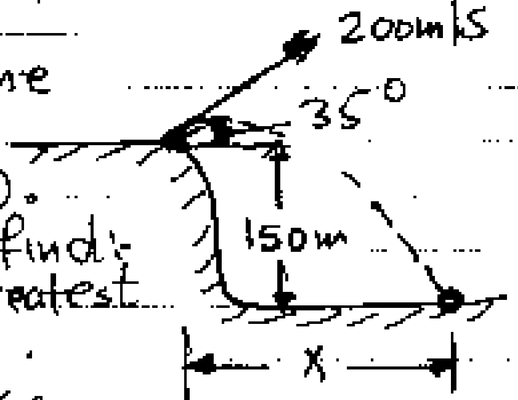
Solving this quadratic equation for ($\tan\alpha$), we have:

$\tan\alpha = 0.565 \Rightarrow \alpha = 29.5°$ and

$\tan\alpha = 2.75 \Rightarrow \alpha = 70°$

ex.10

A projectile is fired from the edge of a (150m) cliff, with an ($V_i = 200$ m/s) at ($\theta = 35°$). Neglecting air resistance, find:- (a) the value of (x), (b) the greatest elevation above the ground reached by the projectile. Use $g = 9.81$ m/s2



Soln

Choosing the origin at the gun, we have:-

$V_{yi} = V_i \sin 35 = (200) \sin 35 = 114.71$ m/s

$V_{yf} = 114.71 - gt$ --- (1)

$y_f = 114.71\,t - 4.9t^2$ --- (2)

$V_{yf}^2 = V_{yi}^2 - 2g(y_f - y_i) \Rightarrow V_{yf}^2 = 13158.38 - 19.62\,y_f$ --- (3)

$V_{xi} = V_i \cos 35 \Rightarrow V_{xi} = 163.83$ m/s

$x_f = x_i + V_{xi}t + \frac{1}{2}a_x t^2 \Rightarrow V_{xf} = 163.83\,t$ -- (4)

when the projectile strikes the ground, we have:- $y_f = -150$ m substitute in eq (2)

$-150 = 114.71\,t - 4.9t^2$ or $t^2 - 23.41\,t - 30.61 = 0$

$t = 24.65$ sec

$x_f = 163.83(24.65) \Rightarrow x_f = 4038.41$ m

At greatest elevation, $V_{yf} = 0$, sub in equ (3)

$0 = 13158.38 - 19.62\,y_f \Rightarrow y_f = h = 670.66$ m

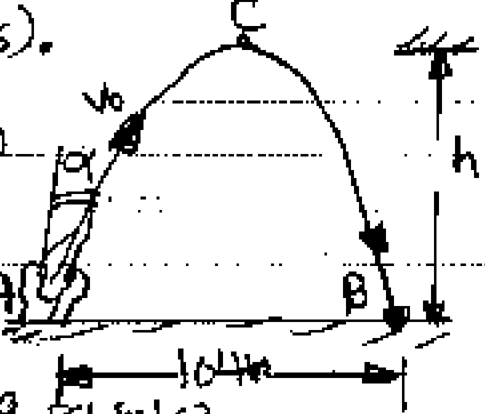
or $h = y = \frac{V_i^2 \sin^2\theta}{2g} = \frac{(200)^2 (\sin 35)^2}{2 \times 9.81} = 670.72$ m

$x_f = R = \frac{V_i^2 \sin 2\theta}{g} = \frac{(200)^2 (\sin 70)}{9.81} = 3831.57$ m

The first solution is more correct and accurate

ex.11

A model rocket is launched from point (A) with an initial velocity ($V_o = 86$ m/s). If the rocket is at (104m) from (A) determine (a) the angle (α) that (V_o) forms with the vertical (b) the maximum height (h) reached by the rocket, (c) the duration of flight. Take: $g = 9.81$ m/s2.



(Solu)

$V_{xi} = 86 \sin\alpha$, $V_{yi} = 86 \cos\alpha$

$X_B = R = 104$ m , $t_B = 2t_C$, $V_{yB} = 0$

$$R = \frac{V_o^2 \sin 2\alpha}{g} \Rightarrow \alpha = \sin^{-1}\left(\frac{Rg}{V_o^2}\right) \Big| 2$$

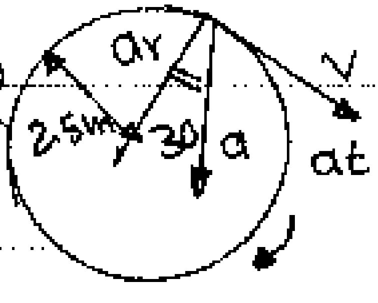
$$\alpha = \sin^{-1}\left[\frac{104 * 9.81}{(86)^2}\right] \Big| 2 \Rightarrow \alpha = 3.96^\circ$$

$$h = \frac{V_o^2 \cos^2\alpha}{2g} = \frac{(86)^2 \cos^2 3.96}{2 * 9.81} \Rightarrow h = 375.16\text{m}$$

$$t_B = \frac{2 V_o \cos\alpha}{g} = \frac{2 * 86 * \cos 3.96}{9.81} \Rightarrow \cancel{t_B = t} = 17.49\text{sec}$$

(EX. 12)

The total acceleration of a particle moving clockwise in a circle of (2.5m) radius at a certaint instant of time. is ($a = 15$ m/s2). At this instant:- find (a) the radial acceleration, (b) the speed of the particle, and (c) its tangential acceleration.



(solu)

(a) $a_r = a \cos 30 = (15)(0.866) \Rightarrow a_r = 13$ m/s2.

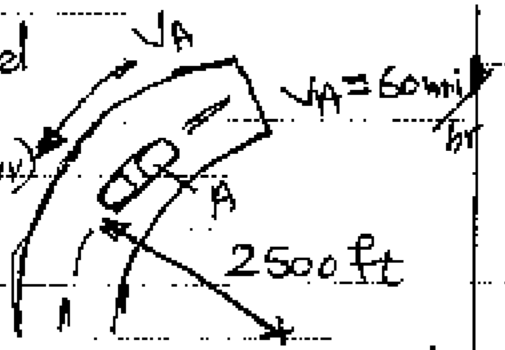
(b) $a_r = V^2/r \Rightarrow V = \sqrt{(a_r)(r)} = \sqrt{(13)(2.5)} \Rightarrow V = 5.7$ m/s

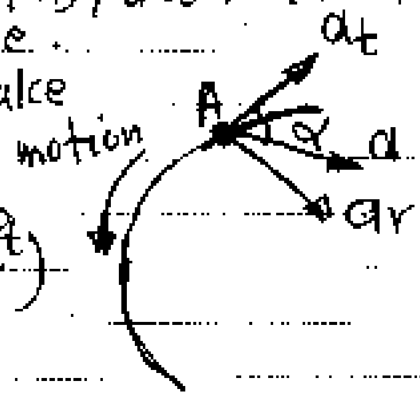
(c) $a_t = a \sin 30 = (15)(0.5) \Rightarrow a_t = 7.5$ m/s2

(or) $a_t = \sqrt{a^2 - a_r^2}$

ex. 13

A motorist is traveling on a curved section of highway of radius (2500 ft) at the speed of (60 mi/hr). The motorist suddenly applies the brakes, causing the automobile to slow down at a constant rate. Knowing that after (8 s) the speed has been reduced to (45 mi/hr), determine the acceleration of the automobile immediately after the brake have been applied.





Sol.

$$V_1 = 60\ mi/h = \left(\frac{60\ mi}{hr}\right)\left(\frac{5280\ ft}{1\ mi}\right) * \left(\frac{1\ h}{3600\ s}\right)$$

$$V_1 = 88\ ft/s$$

$$V_2 = 45\ mi/h \Rightarrow V_2 = 66\ ft/s$$

$$a_t = \frac{\Delta V}{\Delta t} = \frac{(66-88)}{8} \Rightarrow a_t = -2.75\ ft/s^2$$

$$a_r = \frac{V^2}{r} = \frac{(88)^2}{2500} \Rightarrow a_r = 3.10\ ft/s^2$$

$$\tan\alpha = \frac{a_r}{a_t} = \frac{3.10}{-2.75} \Rightarrow \alpha = 48.4^\circ$$

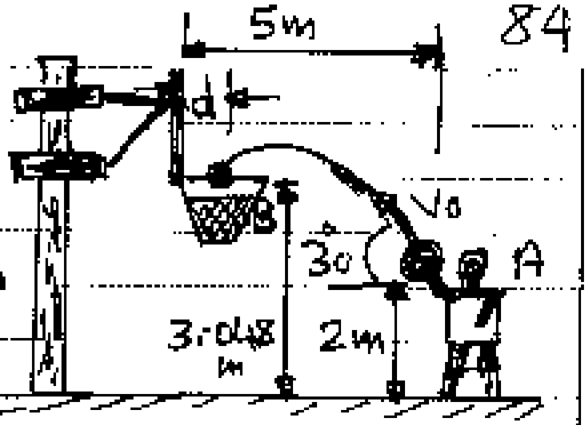
$$a = \sqrt{a_r^2 + a_t^2} = \sqrt{(3.10)^2 + (-2.75)^2} \Rightarrow a = 4.14\ ft/s^2$$

ex. 14

The basketball player shoots when she is (5m) from the backboard. Knowing that the ball has an initial velocity (V_0) at an angle of $(30)^\circ$ with the horizontal, determine the value of (V_0) when (d) is equal to (a) 228 mm, (b) 430 mm.

Take: $g = 9.81\ m/s^2$

Solu



$V_{xo} = V_o \cos 30 = 0.866 V_o$

$V_{yo} = V_o \sin 30 \Rightarrow V_{yo} = 0.5 V_o$

$a_x = 0 \; , \; a_y = -g$

ⓐ $X_i = 0 \; , \; X_f = 5 - 0.228 = 4.772 m$

$4.772 = 0 + 0.866 V_o t + 0$

$V_o t = 5.51$ ---- (1)

$y_f = 3.048 - 2 = 1.048 m \; , \; y_i = 0$

$1.048 = 0 + 0.5 V_o t - \frac{1}{2} \times 9.81 t^2$ --- ② sub eq ① in ②

$t = 0.59$ sec

$V_o = 5.51 / 0.59 \Rightarrow V_o = 9.34$ m/s

ⓑ $X_f = 5 - 0.430 \Rightarrow X_f = 4.57$ m

$4.57 = 0 + 0.866 V_o \Rightarrow V_o t = 5.277$ --- ③

$1.048 = 0 + 0.5 V_o t - 4.905 t^2$ ---- ④

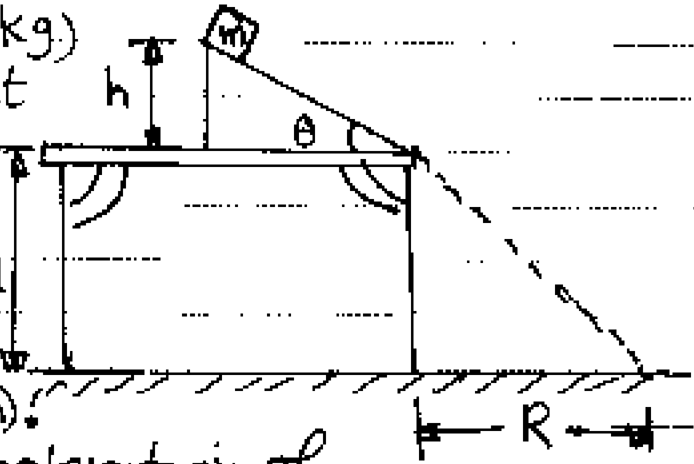
subs equ ③ in equ ④

$t = 0.569$ sec

$V_o = 5.277 / 0.569 \Rightarrow V_o = 9.27$ m/s

EX 15

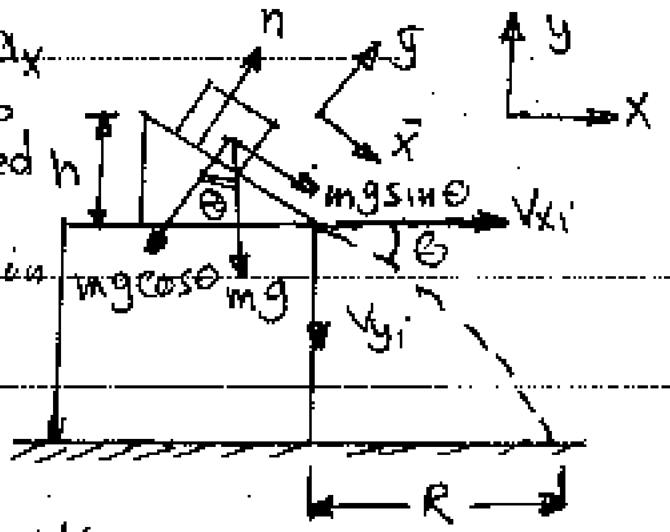
A block of mass $(m = 2 kg)$ is released from rest at $(h = 0.5 m)$ above the surface of a table, at the top of a $(\theta = 30°)$ incline. The frictionless incline is fixed on a table of height $(H = 2 m)$.



ⓐ Determine the acceleration of the block as it slides down the incline. ⓑ What is the velocity of the block as it leaves the incline? ⓒ How far from the table will the block hit the floor? ⓓ How much time has elapsed between when the block is released and when it hits the floor?

(e) Does the mass of the block affect any of the abov calculations?

(Solu)



(a) $\Sigma F_x = mg\sin\theta = m a_x$

$a_x = g\sin\theta = 9.80 \sin 30$

$a_x = 4.90 m/s^2 \rightarrow$ inclined plane

(b) For constant acceleration

$V_{xf}^2 = U_{xi}^2 + 2a_x(x_f - x_i)$

$x_i = 0 , x_f = \frac{h}{\sin 30} = 1m$

$V_{xi} = 0$

$V_{xf}^2 = 0 + 2\times 4.9\ (1-0)$

$V_{xf} = 3.13 m/s = V_i$

(c) For elliptical path

$V_{yi} = V_i \sin 30 \Rightarrow V_{yi} = -1.565 m/s$

$y_f = y_i + V_{yi}t + \frac{1}{2}a_y t^2$

$-2 = 0 + (-1.565)t - \frac{1}{2}(9.80)t^2 \Rightarrow 4.9t^2 + 1.565t - 2 = 0$

$t_2 = 0.5$ sec

$V_{xi} = V_i \cos 30 = (3.13)\cos(-30) = 2.71 m/s$

$V_{xf} = U_{xi} + a_x t \Rightarrow 0 = (2.71) + a_x(0.5) \Rightarrow a_x = -5.42 m/s^2$

$R = x_f = 0 + (2.71)(0.5) + \frac{1}{2}(-5.42)(0.5)^2$

$R = 0.6775 m$

(d) For the first plane

$V_{xf} = U_{xi} + a_x t \Rightarrow t = \frac{V_{xf} - U_{xi}}{a_x} = \frac{(3.13-0)}{4.9}$

$t_1 = 0.638$ sec

Total time $(t) = t_1 + t_2 = 0.638 + 0.5 \Rightarrow t = 1.138$ sec

(ex.16)

A fire man (50 m) away from a burning building directs a stream of water from a ground level fire hose at an angle of (30°) above the horizontal. If the speed of the stream as it leaves the hose is

(40m/s), at what height the stream of water strike the building. Take: $g = 9.80\, m/s^2$

Solu

$V_{0x} = V_0 \cos 30 = (40 m/s) \cos 30 = 34.6 m/s$

$V_{0y} = V_0 \sin 30 = (40 m/s) \sin 30 = 20 m/s$

$t = \Delta x / V_{0x} = 50m / 34.6 m/s = 1.44 sec$

$h = \Delta y = V_{0y} t - \frac{1}{2} a t^2 = (20 m/s)(1.44 s) - \frac{1}{2}(9.80 m/s^2)(1.44 s)^2$

$h = 18.6 m$

ex.17

An arrow is shot straight up in the air at an initial speed of (15m/s). After how much time is the arrow heading downward at a speed of (8m/s)?

(a) 0.714 s (b) 1.24 s (c) 1.87 s (d) 2.35 s (e) 3.22 s

Take $g = 9.8\, m/s^2$.

solu

$V_0 = 15 m/s$, $V_f = -8 m/s$, $g = -9.80 m/s^2$

$\Delta t = \frac{\Delta V}{a} = \frac{V_f - V_i}{-g} = \frac{(-8-15) m/s}{-9.80 m/s^2} = 2.35 sec$

Choice (d) is correct.

ex.18

A projectile is projected at (45°) with horizontal. Show that the horizontal range is (4) times the maximum height.

Solu

$R_{max} = \frac{V^2}{g}$ at $\theta = 45°$ and $h = \frac{V^2 \sin^2\theta}{2g}$

$h_{max} = \frac{V^2}{2g} (\sin 45)^2 = \frac{V^2}{2g} (0.707)^2 = \frac{V^2}{2g} * \frac{1}{2} = \frac{V^2}{4g}$

$\therefore h_{max} = \frac{1}{4} R$ or $R = 4 h_{max}$

ex. 19

An aircraft executes a horizontal loop of a radius (1 Km), with a steady speed of (900 km/h). Compute its centripetal acceleration with acceleration due to gravity. Take $g = 9.80 m/s^2$.

Solu.

$V = 900 km/h * \frac{1000}{3600} = 250 m/s$, $r = 1 Km = 1000 m$

$a_c = \frac{V^2}{r} = \frac{(250)^2}{1000} = 62.5 m/s^2$

$a_c / g = 62.5 / 9.80 = 6.38$

ex. 20

A ball is projected with a velocity of ($40\sqrt{2}$ m/s) at an angle of (45°). Find the position and velocity of the ball after (2) second.

Solu.

$V_i = 40\sqrt{2} m/s$, $\theta = 45°$

$V_{xi} = V_i \cos\theta = 40\sqrt{2} \cos 45 = 40 m/s$

$V_{yi} = V_i \sin\theta = 40\sqrt{2} \sin 45 = 40 m/s$

After (2) sec

$V_{yf} = V_{yi} + a_y t = 40 - (9.80)(2) = 20.4 m/s$

$V_{xf} = V_{xi}$ because ($a_x = 0$)

$V = \sqrt{V_x^2 + V_y^2} = \sqrt{40^2 + (20.4)^2} = 44.90 m/s$

$\tan\theta = V_y / V_x = \frac{20.4}{40} \Rightarrow \theta = 27.02°$

position

$y_f = 0 + (40)(2) - \frac{1}{2}(9.80)(2)^2$

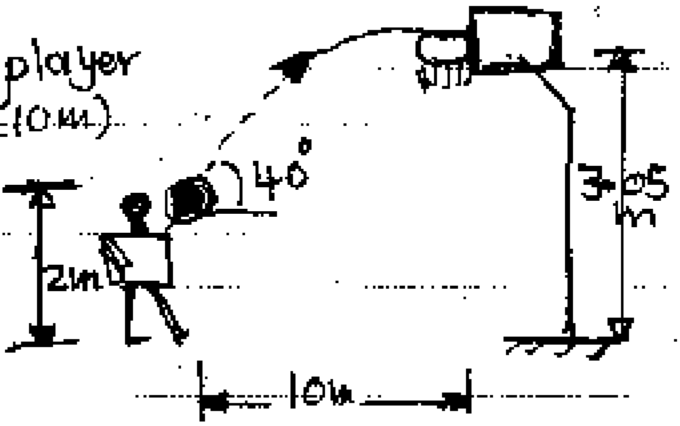
$= 60.4 m$

$X_f = 0 + 40(2) + 0 = 80 m$

ex.21

A (2m) tall basket ball player is standing on the floor (10m) from the basket. If he shoots the ball at a (40°) angle, at what speed he throw the basketball so that it goes through the loop.

Take $g = 9.80\ m/s^2$



solu

$V_{xi} = V_i \cos\theta = V_i \cos 40 = 0.766 V_i$

$t = \frac{\Delta x}{0.766 V_i} = \frac{10}{0.766 V_i}$, $t = \frac{\Delta x}{V_{xi}}$

$t = \frac{13.1}{V_i}$, $\Delta y = y_f - y_i = 3.05 - 2 = 1.05 m$

$1.05 = V_i \sin 40 * \frac{13.1}{V_i} - \frac{1}{2}(9.80)\left(\frac{13.1}{V_i}\right)^2$

$V_i = 10.7 m/s$

---

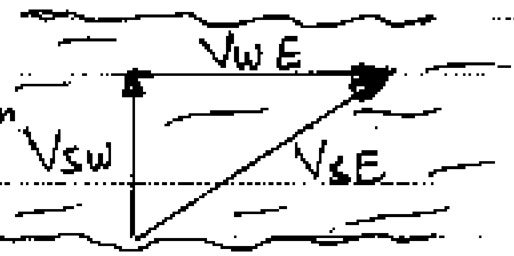
ex.22

A river has a steady speed of (0.5 m/s). A student swims upstream a distance of (1 Km) and swims back to the starting point. ⓐ If the student can swim at a speed of (1.2 m/s) in still water, how long does the trip take? ⓑ How much time is required in still water for the same length swim? ⓒ Why does the swim take longer when there is a current?

solu $\vec{V}_{SE} = \vec{V}_{SW} + \vec{V}_{WE}$

$\vec{V}_{WE} = 0.5 m/s$, $\vec{V}_{SW} = 1.2 m/s$ downstream

$\vec{V}_{SW} = -1.20 m/s$ upstream



$(\vec{V}_{SE})_{up} = \vec{V}_{WE} + (\vec{V}_{SW})_{up} = 0.5 + (-1.2) = -0.7\ m/s$

and

$(\vec{V}_{SE})_{down} = \vec{V}_{WE} + (\vec{V}_{SW})_{down} = 0.5 + 1.2 = 1.7\ m/s$

The distance for each leg of trip (relative to Earth)

(a) $(d = 1\ Km = 1000\ m)$.

$$t_{up} = \frac{d}{(V_{SE})_{up}} = \frac{1000}{0.7} = 1.43 \times 10^{3}\ sec$$

$$t_{down} = \frac{d}{(V_{SE})_{down}} = \frac{1000}{1.7} = 5.88 \times 10^{2}\ sec$$

Total time $= (1.43 \times 10^{3} + 5.88 \times 10^{2}) = 2.02 \times 10^{3}\ sec$

(b)

If the water still, $|\vec{V}_{WE}| = 0$, so that $|\vec{V}_{SE}| = |\vec{V}_{SE}|_{up} = |\vec{V}_{SE}|_{down} = 1.2\ m/s$ so that $t_{leg} = \frac{d}{|\vec{V}_{SE}|} = \frac{1000}{1.2}$
$= 8.33 \times 10^{2}\ sec$ and
$t_{total} = 2 t_{leg} = 1.67 \times 10^{3}\ sec$

(c) The time savings going downstream with the current is always less than the extra time required to go the same distance against the current.

(ex.23)

A jet plane traveling horizontally at (100 m/s) drops a rocket from a certain height. The rocket accelerating at (20 m/s) in the x-direction while falling under the influence of gravity in y-direction when the rocket has fallen (1 km) find (a) its velocity in the y-direction (b) its velocity in the x-direction and (c) the magnitude and direction of its velocity. Neglect air drag. Take: $g = 9.8\ m/s^2$.

(Solu)

(a) $V_{yf}^2 = V_{yi}^2 - 2g\Delta y = 0 - 2(9.80)(-1 \times 10^{3} - 0)$

$V_{yf} = -1.40 \times 10^{2}\ m/s$

ⓑ $V_{yf} = V_{yi} + a_y t \Rightarrow -1.4 \times 10^2 = 0 - (9.80)(t)$

$t = 14.3\ s$

$V_{xf} = V_{xi} + a_x t = 100 + (20)(14.3) \Rightarrow V_{xf} = 386\ m/s$

ⓒ $V_f = \sqrt{V_{xf}^2 + V_{yf}^2} = \sqrt{(386)^2 + (-140)^2} \Rightarrow V_f = 411\ m/s$

$\theta = \tan^{-1}\left(\frac{-140}{386}\right) \Rightarrow \theta = -19.9°$

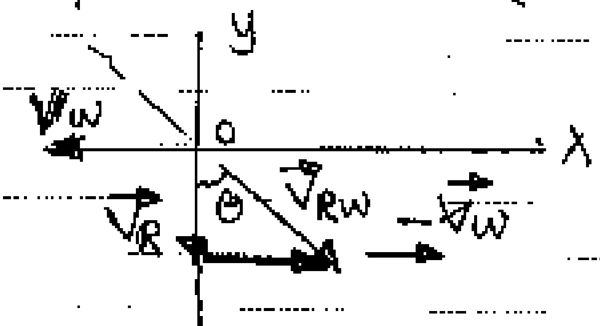
---

ex.24

Rain is falling vertically with a speed of (35 m/s). A women rides a bicycle with a speed of (12 m/s), in east to west direction. what is the direction in which she hold her umbrella? what is the velocity of rain with respect to women?

Solu: velocity of rain with respect to women is given by

$\vec{V}_{RW} = \vec{V}_R - \vec{V}_W$



$\tan\theta = V_W / V_R = 12/35 \Rightarrow \theta = 19°$

$V_{RW} = -12\hat{i} + 35\hat{j}$

$V_{RW} = \sqrt{(-12)^2 + (35)^2} \Rightarrow V_{RW} = 37\ m/s$

---

ex.25

A record of travel along a straight path is as follows

1- Start from rest with a constant acceleration of $(2.77\ m/s^2)$ for (15) s.

2- Maintain a constant velocity for the next (2.05) min

3- Apply a constant negative acceleration $(-9.47\ m/s^2)$ for (4.39) s

ⓐ- what was the total displacement for the trip?

(b) what where the average speeds for legs 1, 2, and 3 of the trip, as well as for the complete trip?

Solu

$V = V_0 + at = 0 + 2.77 * 15 = 41.6$ m/s at the end of first trip, It is constant during the second interval and the initial velocity of the third interval.

Duration of the second interval $= 2.05 * 60 = 123$ s.

$\Delta X = V_0 t + \frac{1}{2} a t^2$ and $(\Delta X)_{total} = (\Delta X)_1 + (\Delta X)_2 + (\Delta X)_3$

(a) $(\Delta X)_t = (0 + \frac{1}{2} \times 2.77 \times 15^2) + (41.6 * 123 + 0) + (41.6 \times 4.39) + \frac{1}{2}(-9.47)(4.39)^2$
$= 6.52$ Km

(b) $\bar{V_1} = \frac{\Delta X_1}{\Delta t_1} = 312\text{m}/15 = 20.8$ m/s

$\bar{V_2} = 5.12 \times 10^3 / 123 = 41.6$ m/s

$\bar{V_3} = \frac{91.4}{4.39} = 20.8$ m/s and $\bar{V}_{total} = \frac{\Delta X_{total}}{t_{total}}$

$\bar{V}_{total} = \frac{5.52 \times 10^3 \text{m}}{(15 + 123 + 4.39)} = 38.8$ m/s

---

Ex: 26

A small bag is released from a helicopter that is descending steadily at (1.5 m/s). After (2 s). (a) What is the speed of the mailbag? (b) what far is it below the helicopter? (c) what are answers to parts (a) and (b) if the helicopter is rising steadily at (1.5 m/s2)?

Solu

(a) $V_{bag} = V_0 + at = -1.5 + (-9.80)(2) = -21.1$ m/s

(b) Displacement of the bag is

$(\Delta y)_{bag} = \frac{1}{2}(V + V_0)t = \left[\frac{-21.1 + (-1.50)}{2}\right](2) = -22.6$ m

During this time, the helicopter moving downward with constant velocity undergoes a displacement of

$(\Delta y)_{heli} = V_0 t + \frac{1}{2} a t^2 = (-1.5\ m/s)(2\ s) + 0 = -3\ m$

The distance separating the package and the helicopter at this time is:-

$d = |(\Delta y)_p - (\Delta y)_h| = |-22.6 - (-3)| = 19.6\ m$

(c)

Here $(V_0)_{bag} = (V_0)_h = +1.5\ m/s$ and $a_{bag} = 9.80\ m/s^2$ while $a_{heli} = 0$

After (2 sec), the velocity of the mailbag is

$V_{bag} = 1.5\ m/s + (-9.80)(2) = -18.1\ m/s$ and its speed is $|V_{bag}| = 18.1\ m/s$

In this case the displacement of the helicopter during the (2 sec) is

$(\Delta y)_{heli} = (+1.5\ m/s)(2\ s) + 0 = +3\ m$

The mailbag has a displacement of

$(\Delta y)_{bag} = \frac{V_{bag} + V_0}{2} t = \left(\frac{-18.1 + 1.5}{2}\right) 2 = -16.6\ m$

The distance separating the package and the helicopter is

$d = |(\Delta y)_p - (\Delta y)_h| = |-16.6\ m - (+3\ m)| = 19.6\ m$

(ex-27)

A rocket moves straight upward, starting from rest with an acceleration of $(29.4\ m/s^2)$. It run out of fuel at the end of (4) second and continues to coast upward, reaching a maximum height before falling back to earth. (a) Find the rocket's velocity and position at the end of (4 sec). (b) Find the maximum height the rocket reaches. (c) Find the velocity at the instant before the rocket crashes on the ground. Take: $g = 9.80\ m/s^2$

Sol) ⓐ case 1

$V = V_0 + at$ ---(1), $y = y_0 + V_0 t + \frac{1}{2}at^2$ ---(2)

$V_b = 0 + (29.4)(4) = 117.6 \simeq 118\ m/s$

$y_b = \frac{1}{2}(29.4)(4)^2 \simeq 235\ m$

ⓑ case ② $g = -9.80\ m/s^2$, $V_0 = V_b = 118\ m/s$

$y_0 = y_b = 235\ m$

$V = V_b + at \Rightarrow V = 118 - 9.8t$ ---(3)

$y_{max} = 235 + 118t - 4.9t^2$ ---(4)

substitute ($V=0$) in equ ③ (at the maximum height, velocity is zero) $\Rightarrow t = 12\ sec$, $y_{max} = 945\ m$

ⓒ

In equ ④ setting (y) $= 0$

$0 = 235 + 118t - 4.9t^2 \Rightarrow t = 25.9\ sec$

$V = (-9.80\ m/s^2)(25.9\ s) + 118\ m/s$

$= -136\ m/s$

ex.28

A ball is thrown upward from the ground with an initial speed (25 m/s) at the same instant, another ball is dropped from a building (15 m) high. After how long will the balls be at the same height?

Sol)

The falling ball moves a distance of (15m − h) before they meet, where (h) is the height above the ground where they meet.

$\Delta y = V_0 t + \frac{1}{2}at^2 \Rightarrow -(15m - h) = 0 - \frac{1}{2}gt^2$ or

$h = 15m - \frac{1}{2}gt^2$

Rising ball, $\Delta y = h = (25\ m/s)t - \frac{1}{2}gt^2$

$(25\ m/s)t - \cancel{\frac{1}{2}gt^2} = 15m - \cancel{\frac{1}{2}gt^2} \Rightarrow t = 0.60\ sec.$

ex.29

A projectile is launched with an initial speed of (60m/s) at an angle of (30°) above the horizontal. The projectile lands on a hillside (4 sec) later. (a) what is the projectile's velocity at the highest point of its trajectory? (b) what is the straight-line distance from where launched to where it hit its target? Take: $g = 9.80 \, m/s^2$

Solu

(a) At the highest point, $V_y = 0$

$V_x = V_{ox} \cos\theta = (60 m/s) \cos 30 = 52 m/s$

(b) $\Delta x = V_{ox} t = (52 m/s)(4 s) = 208 m$

$\Delta y = (V_o \sin\theta) t - \frac{1}{2} g t^2 = (60 m/s) \sin 30 (4) - \frac{1}{2} * 9.80 (4)^2$

$\Delta y = 41.6 m$

The straight line distance $(d) = \sqrt{\Delta x^2 + \Delta y^2} = \sqrt{(208)^2 + (41.6)^2}$

$d = 212 m$

ex.30

A home run is hit in such a way that the base ball just clears a wall (21m) high located (130m) from home plate. The ball is hit at angle of (35°) to the horizontal. (a) Find the initial speed of the ball. (b) Find the time it takes the ball to reach the wall. (c) Find the velocity components and the speed of the ball when it reaches the wall. (Assume the ball is hit at a height of 1m above the ground).

Solu

(a) The time for the ball to reach the fence is:-

$t = \Delta x / V_{ox} = 130 m / V_o \cos 35 = \frac{159 m}{V_o}$

At that time the ball must be $\Delta y = 21 - 1 = 20 m$ above its launch position, so, $\Delta y = V_{oy} t + \frac{1}{2} a t^2$

$$20m = (V_0 \sin 35)\left(\frac{159m}{V_0}\right) - \frac{1}{2} \times (9.80 m/s^2)\left(\frac{59}{V_0}\right)^2$$

Or

$$(159m)\sin 35 - 20 = \frac{\frac{1}{2}(9.8m/s^2)(159m)^2}{V_0^2}$$

$$V_0 = 42 m/s$$

(b) $t = 159m / 42m/s = 3.8 sec.$

(c) $V_x = V_{0x} = (42m/s)\cdot\cos 35 = 34 m/s$

$V_y = V_{0y} + a_y t = (42m/s)\sin 35 - (9.80m/s^2)(3.8s)$
$= -13 m/s$

$$V = \sqrt{V_x^2 + V_y^2} = \sqrt{(34 m/s)^2 + (-13 m/s)^2} = 36 m/s$$

ex.31

A person takes a trip, driving with a constant speed of (89.5 km/h), except for (22 min) rest stop. If the person's average speed is (77.8 km/h), how much time is spent on the trip and how far does the person travel?

Solu

The total time of trip is ($t_{total} = t_1 + 22 min = t_1 + 0.367 h$)

$\Delta X = v_1 t_1 \Rightarrow (89.5 km/h) t_1 = (77.8 km/h)(t_1 + 0.367h)$
$= (77.8 km/h) t_1 + 28.5 km$

Or

$(89.5 - 77.8) km/h (t_1) = 28.5 km$

From which, $t_1 = 2.44 hr$

$t_{total} = t_1 + 0.367 hr = 2.81 hr.$

(b) $\Delta X = v_1 t = V t_{total} = (77.8 km/h)(2.81 hr)$
$= 219 km$

Lecture
(7)

Chap. 5

The Laws of Motion

When several forces act simultaneously on an object, it's accelerates if the net force acting on it is not equal to (zero). The (net force) acting on an object is defined as (the vector sum of all forces acting on the object). we sometimes refer to the net force as the (total force), (resultant force), or the (unbalanced force).

If the net force exerted on an object is (zero), the acceleration of the object is (zero) and its velocity remains constant.

When the velocity is constant (including when the object is at rest), the object is said to be in equilibrium.

The forces may be: 1- (contact forces) that is involve physical contact between two objects. Examples, the force exerted by gas molecules on the walls of a container, the force exerted by your feet on the floor, stretching of coiled spring by pulling it, when a football is kicked.

2- (Field forces): they do not involve physical contact between two objects but act through empty space. Examples: the gravitational force of attraction between two objects that keeps objects bound to Earth and the planets in orbit around the sun, the force a bar magnet exerts on a piece of iron, the electric force that one electric charge exerts on another.

Because force is a vector we use the symbol ($\vec{F}$) and the rules of vector addition to obtain the net force on an object.

Newton's First Law and Inertial Frames

Newton's first law of motion sometimes called the (law of inertia) defines a special set of reference frames called (inertial frames). This law can be stated as (If an object does not interact with other objects, it is possible to identify a reference frame in which the

object has zero acceleration).
such a reference frame is called an (inertial frame of reference).
(Any reference frame that moves with constant velocity relative to an inertial frame itself is an inertial frame).
A reference frame that moves with constant velocity relative to the distant stars is the best approximation of an inertial frame. The Earth is not really an inertial frame because of its orbital motion around the sun and its rotational motion about its own axis, both of each result in (centripetal accelerations).
These accelerations are small compared with (g) and can often be neglected. For this reason Earth is considered as an inertial frame.
Another statement of Newton's first law is that an object at rest remains at rest and an object in motion continues in motion with a constant velocity (that is with a constant speed on a straight line).
Newton's first law does not say what happens for an object with zero net force, that is multiple forces that cancel.
The tendency of an object to resist any attempt to change its velocity is called (inertia).

Mass

It is defined as "the resistance an object exhibits to changes in its velocity". The greater the mass of an object, the less that object accelerates under the action of a given applied force.
Mass is a scalar quantity. Mass and weight are two different quantities. "The weight of an object is equal to the magnitude of the gravitational force exerted on the object and varies with location". For example a person who weighs (180 lb) on the Earth weighs only about (30 lb) on the Moon. On the other hand.

the mass of an object is the same everywhere.

When a force acting on an object of mass (m_1) produces an acceleration (a_1) and the same force acting on another object of mass (m_2) produces an acceleration (a_2). The ratio of the two masses is defined as the inverse ratio of the magnitudes of the accelerations produced by the force:-

$$\boxed{m_1/m_2 = a_2/a_1}$$

Newton's Second Law

The acceleration of an object is directly proportional to the force acting on it. The magnitude of the acceleration of an object is inversely proportional to its mass.

Force does not cause motion but it is the cause of change in motion, as measured by acceleration.

We can relate mass, acceleration, and force through the following mathematical statement of (Newton's second law):-

$$\sum \vec{F} = m\vec{a} \quad \text{(Newton's second law)}$$

This equation is valid only when the speed of the object is much less than the speed of light.

The (net force $\sum F$) on an object is the vector sum of all forces acting on the object. There may be many forces acting on the object, but there is only one acceleration.

$$\sum F_x = m\,a_x \;,\; \sum F_y = m\,a_y \;,\; \sum F_z = m\,a_z \quad \text{(component form)}$$

The unit of force in (SI) system is (newton). Newton is defined as: [the force that, when acting on an object of mass (1 kg), produces an acceleration of ($1\,m/s^2$)]. ($1\,N = 1\,kg \cdot m/s^2$).

In the U.S. customary system, the unit of force is the (pound). pound is defined as (the force that, when acting on a (1 slug) of mass produces an acceleration of ($1\,ft/s^2$)].

$1\,lb = 1\,slug \cdot ft/s^2$ and $1\,N \simeq \frac{1}{4}\,lb$.

The Gravitational Force and weight

Gravitational force (F_g) is the "attractive force exerted by the Earth on an object and it is directed toward the center of the Earth", it is magnitude is called the weight of the object. Applying Newton's second law $\sum \vec{F} = m\vec{a}$ to a freely falling object of mass (m) with $\vec{a} = \vec{g}$ and $\sum \vec{F} = \vec{F}_g$ we obtain

$$\vec{F}_g = m\vec{g}$$

Because it depends on (g), weight varies with geographic location. Objects weigh less at higher altitudes than at sea level. The mass in the above equation is called (gravitational mass). The Kilogram is not a unit of weight, it is a unit of mass.

Newton's Third Law

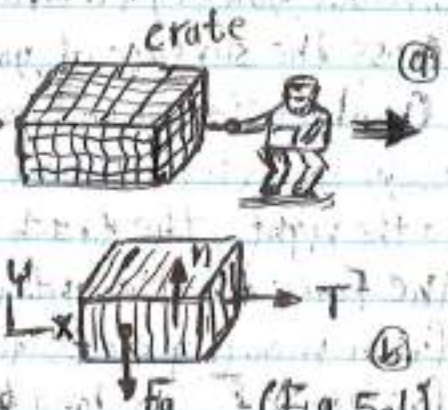
"If two objects interact, the force (F_{12}) exerted by object 1 on object 2 is equal in magnitude and opposite in direction to the force (F_{21}) exerted by object 2 on object 1."

$$\vec{F}_{21} = -\vec{F}_{12}$$

The force $\vec{F}_{12}$ is called the (action force) and the force F_{21} is called (reaction force). In all cases they act on different objects and must be of the same type. Example, the force acting on a freely falling projectile is the gravitational force exerted by the Earth on the projectile $\vec{F}_g = \vec{F}_{EP}$ and the magnitude of this force is (mg), The reaction to this force is the gravitational force exerted by the projectile on the Earth $\vec{F}_{PE} = -\vec{F}_{EP}$. The action force $\vec{F}_{EP}$ accelerates the projectile toward the Earth while $\vec{F}_{PE}$ must accelerate the Earth toward the projectile. Because the Earth has such a large mass, its acceleration due to this reaction force is negligibly small. Newton's third law action and reaction forces act on different objects. Two forces acting on the same object, even if they are equal in magnitude and opposite in direction, cannot an action_reaction pair.

objects Experiencing a Net Force

consider a crate being pulled to the right on a frictionless, horizontal surface (Fig 5.1/a). The horizontal force ($\vec{T}$) being applied to the crate acts through the rope. The magnitude of ($\vec{T}$) is equal to the tension in the rope (Fig 5.1/b), $\vec{n}$ is the normal force exerted by the floor on the crate.



(Fig 5.1)

We can now apply Newton's second law in component form to the crate.

$$\sum F_x = ma_x = T \quad \text{or} \quad a_x = \frac{T}{m}$$

No acceleration occurs in the y direction, $\sum F_y = m a_y$ with $a_y = 0$

$\therefore \sum F_y = 0 \Rightarrow n + (-F_g) = 0 \Rightarrow n = F_g$

If $\vec{T}$ is a constant force then the acceleration ($a_x = \frac{T}{m}$) also is constant. The constant acceleration equations of Kinematics can be used to obtain the crate's position (x) and velocity (v_x) as functions of time,

$$v_{xf} = v_{xi} + \left(\frac{T}{m}\right)t \quad \text{and} \quad x_f = x_i + v_{xi}t + \frac{1}{2}\left(\frac{T}{m}\right)t^2$$

$\vec{n}$ (normal force) is not always equal to the magnitude of (F_g)

suppose a book is lying on a table and you push down on the book with a force $\vec{F}$ (Fig 5.2), the book is at rest and therefore not accelerating, $\sum F_y = 0$ which gives, $n - F_g - F = 0$ or $n = F_g + F$. In this example $n > F_g$.

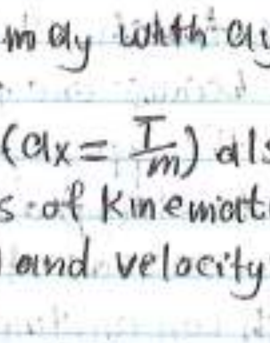


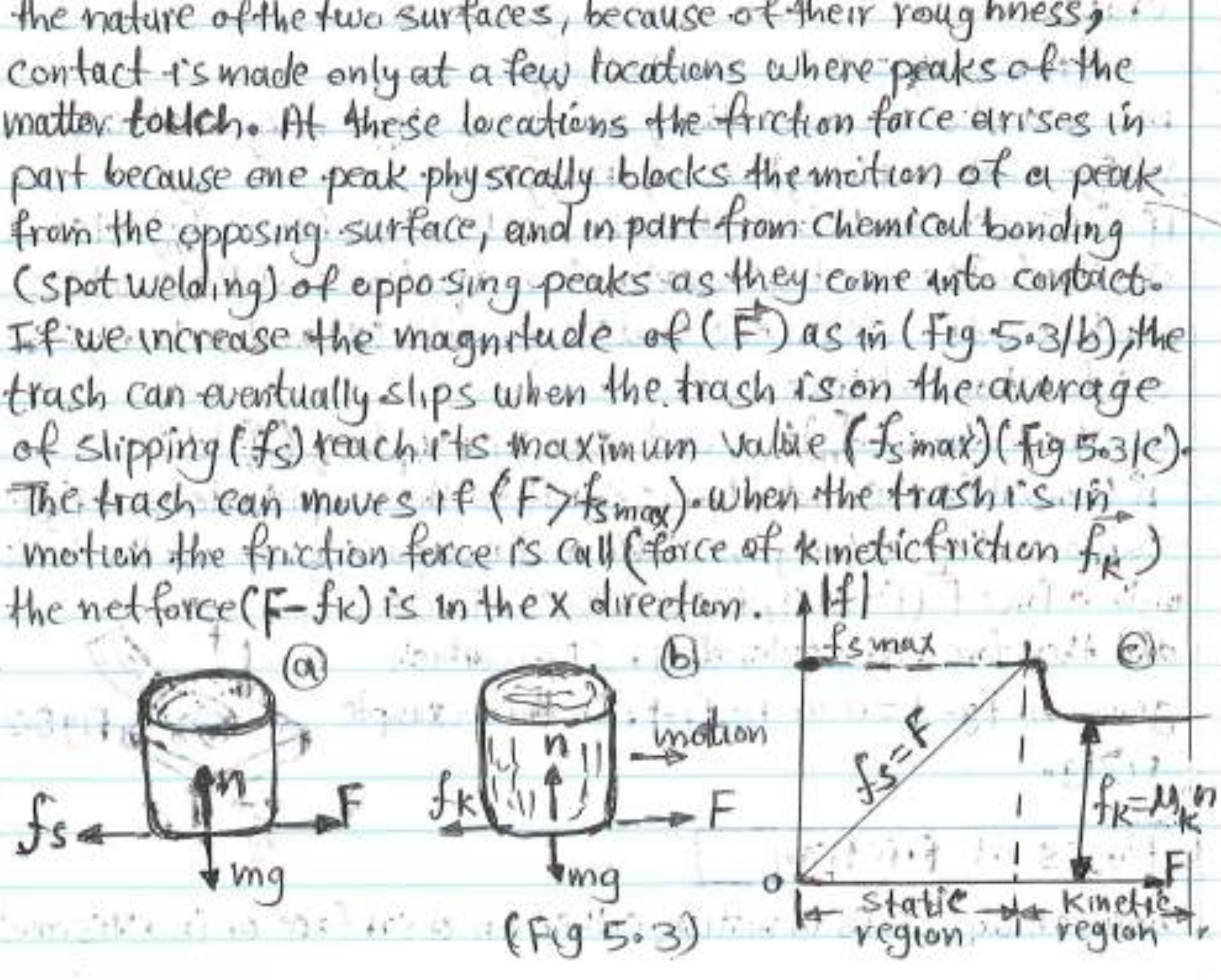
Fig 5.2

Forces of Friction

when an object is in motion either on a surface or in a viscous.

medium such as air or water, there is resistance to the motion because the object interacts with its surroundings. This resistance is a (force of friction).

consider you try to drag a trash can filled with yard clippings across the surface of your concrete patio (Fig 5.3/a). It is a (real surface) not an idealized, frictionless surface. If we apply an external horizontal force ($\vec{F}$) to the trash can acting to the right, the trash can remains stationary if ($\vec{F}$) is small. The Force that counteracts ($\vec{F}$) and keeps the trash can from moving acts to the left and is called the (force of friction) $\vec{f}_s$. As long as the trash can is not moving ($f_s = F$). If F increased, f_s also increases. Friction force arises from the nature of the two surfaces, because of their roughness; contact is made only at a few locations where peaks of the matter touch. At these locations the friction force arises in part because one peak physically blocks the motion of a peak from the opposing surface, and in part from chemical bonding (spot welding) of opposing peaks as they come into contact. If we increase the magnitude of ($\vec{F}$) as in (Fig 5.3/b), the trash can eventually slips when the trash is on the average of slipping (f_s) reach its maximum value ($f_{s\,max}$) (Fig 5.3/c). The trash can moves if ($F > f_{s\,max}$). when the trash is in motion the friction force is call (force of kinetic friction f_k) the net force ($F - f_k$) is in the x direction.



(Fig 5.3)

and produces acceleration to the right, according to the Newton's second law. If $F = f_s$, the acceleration is zero, and the trash can moves to the right with constant speed. If the force is removed, the friction force acting to the left, provids an accleration of the trash can in the x direction and eventually brings it to rest.

Experimental observations

1- The magnitude of the force of static friction between any two surfaces in contact can have the values: $f_s \leq \mu_s n$
where μ_s = coefficient of static friction (it is dimensionless constant).
n = normal force exerted by one surface on the other.
When the surfaces are on the average of slipping $f_s = f_{max} = \mu_s n$, this situation called impending motion.
2- The magnitude of the kinetic friction force acting between two surfaces is:- $f_k = \mu_k n$
where μ_k = coefficient of kinetic friction, it is vary with speed.
3- The values of μ_s and μ_k depend on the nature of the surfaces, but generally $\mu_k < \mu_s$.
4- The direction of the friction force on an object is parallel to the surface with which the object is in contact and opposite to the actual motion (kinetic friction), or the impending motion (static friction) of the object relative to the surface.
5- The coefficient of friction is independent of the area of contact between the surfaces.
6- The equations ($f_s \leq \mu_s n$) and ($f_k = \mu_k n$) are not vector equations. They are relationships between the magnitude of the vectors representing the friction and normal forces. Because the friction and normal forces are perpendicular to each other, the vectors cannot be related by a multiplicative constant.

Examples

Ex-1

A hockey puck having a mass of (0.30 kg) slides on the horizontal, frictionless surface of an ice rink. Two hockey sticks strike the puck simultaneously, exerting the forces on the puck as shown in figure. Determine both the magnitude and direction of the puck's acceleration.

Solu

This problem can be resolved using Newton's second law, we resolve the force vectors into components. The net force acting on the puck in the x direction is:-

$$\Sigma F_x = F_{1x} + F_{2x} = F_1 \cos(-20) + F_2 \cos 60 = (5.0N)(0.940) + (8.0N)(0.500) = 8.7N$$

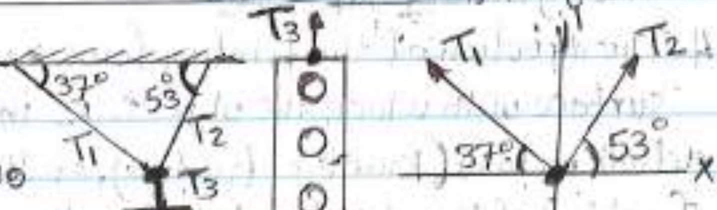
The net force acting on the puck in the y direction is:-

$$\Sigma F_y = F_{1y} + F_{2y} = F_1 \sin(-20) + F_2 \sin 60 = (5.0N)(-0.342) + (8.0N)(0.866) = 5.2N$$

$$a_x = \frac{\Sigma F_x}{m} = \frac{8.7N}{0.30kg} = 29 m/s^2, \quad a_y = \frac{\Sigma F_y}{m} = \frac{5.2N}{0.30kg} = 17 m/s^2$$

$$a = \sqrt{a_x^2 + a_y^2} = \sqrt{(29)^2 + (17)^2} = 34 m/s^2, \quad \theta = \tan^{-1}\frac{a_y}{a_x} = \tan^{-1}\frac{17}{29} = 30^\circ.$$

Ex-2



A traffic light weighing (122N) hangs from a cable tied to two other cables fastened to a support as in (figure a).

The upper cables are not as strong as the vertical cable, and will break if the tension in them exceeds (100N). Will the traffic light remain hanging in this situation, or will one of the cables break?

Solu

Let us assume that the cables do not break so that there is no acceleration of any sort in this problem in any direction.

Because the acceleration of the system is (zero) we know that the net force on the light and the net force on the knot are both zero.
From the free-body diagram for the traffic light (Fig.b)

$\sum F_y = 0 \Rightarrow T_3 - F_g = 0 \Rightarrow T_3 = F_g = 122N$

From the free-body diagram for the knot (fig.c)

$\sum F_x = T_{2x} + T_{1x} = T_2 \cos 53 - T_1 \cos 37 = 0$ ------(1)

$\sum F_y = T_2 \sin 53 + T_1 \sin 37 + (-122N) = 0$ ------(2)

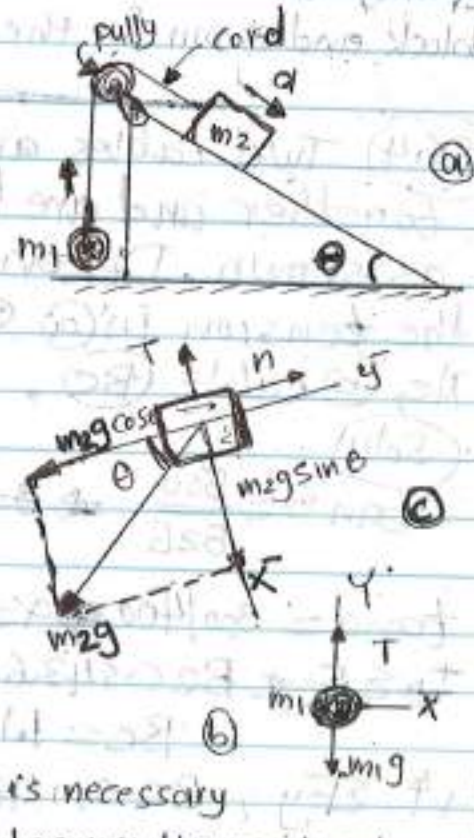
From equ.1 $\Rightarrow T_2 = 1.33 T_1$ substitute in equ.2 to obtain

$T_1 = 73.4N$ and $T_2 = 97.4N$.

Both of these values are less than (100N), so the cables will not break.

Ex3

A ball of mass (m_1) and a block of mass (m_2) are attached by a light weight cord that passes over a frictionless pully of negligible mass (fig a). Find the magnitude of the acceleration of the two objects and the tension in the cord.



Solu

The accelerations of the two objects are the same because they connected by a cord.
Applying Newton's second law in component form to the ball (figb).

① $\sum F_x = 0$, ② $\sum F_y = T - m_1 g = m_1 a_y = m_1 a$

In order for the ball to accelerate upward, it is necessary that $T > m_1 g$. In equ ② (a_y) is replaced by (a) because the acceleration has only (y) component. For the block (fig.c), it is convenient to choose the positive $\bar{x}$ axis along the incline. For consistency with our choice for the ball, we choose the positive direction to be down the incline.

③ $\sum F_{\bar{x}} = m_2 g \sin\theta - T = m_2 a_{\bar{x}} = m_2 a$

(4) $\Sigma F_y = n - m_2 g\cos\theta = 0$

In equ. (3) we replaced (a_x) with (a) because the two objects have accelerations of equal magnitude (a).

From equ. (2): $T = m_1 g + m_1 a$ substitute in equ. (3), we obtain

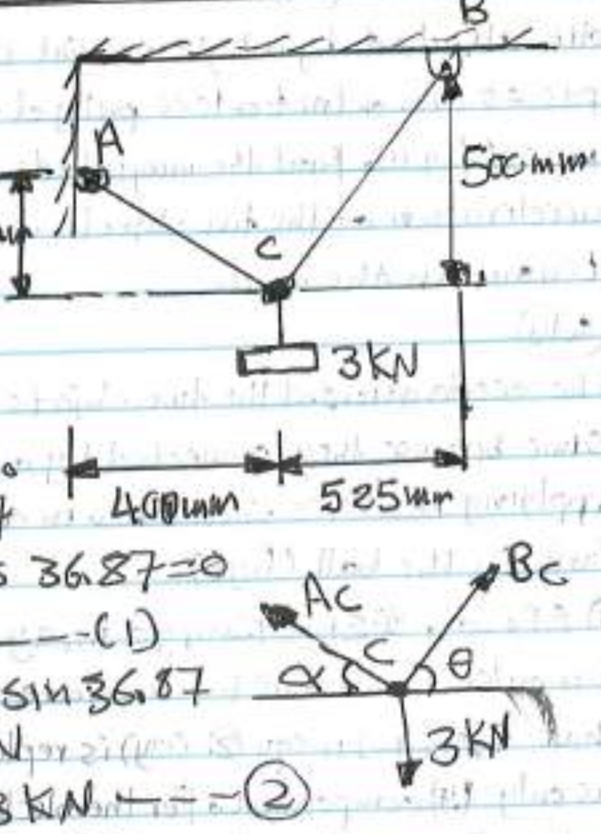
(5) $a = \dfrac{m_2 g\sin\theta - m_1 g}{m_1 + m_2}$ when substitute in equ. (2), we find

(6) $T = \dfrac{m_1 m_2\, g(\sin\theta + 1)}{m_1 + m_2}$

The block accelerates down the incline only if $m_2 \sin\theta > m_1$.

If $m_1 > m_2 \sin\theta$ then the acceleration is up the incline for the block and down for the ball.

(ex.4) Two cables are tied together and are loaded as shown. Determine the tension in (a) cable AC, (b) cable (BC).



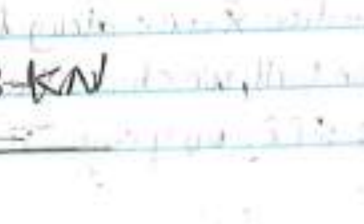
(Solu)

$\tan\theta = \dfrac{500}{525} \Rightarrow \theta = 43.6°$

$\tan\alpha = 300/400 \Rightarrow \alpha = 36.87$

$\overset{+}{\rightarrow} \Sigma F_x = BC\cos 43.6 - AC\cos 36.87 = 0$

$BC = 1.105\, AC$ ---(1)



$+\uparrow \Sigma F_y,\ BC\sin 43.6 + AC\sin 36.87 = 3KN$

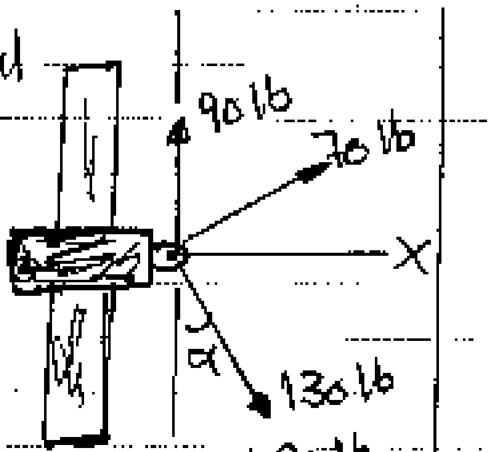
$0.689\, BC + 0.60\, AC = 3KN$ ---(2)

substitute eq (1) in eq (2)

$AC = 2.20\,KN,\ BC = 2.43\,KN$

ex.5

A collar that can slide on a vertical rod is subjected to the three forces as shown. Determine: (a) the value of the angle (α) for which the resultant of the three forces is horizontal, (b) the magnitude of the resultant.



Solu

$\Sigma F_x = R = 70\cos\alpha + 130\sin\alpha$ ---(1)

$\Sigma F_y = 0$, $90 + 70\sin\alpha = 130\cos\alpha$

$90\sec\alpha + 70\tan\alpha = 130$

$81\sec^2\alpha = (13 - 7\tan\alpha)^2$

$81(1+\tan^2\alpha) = (13 - 7\tan\alpha)^2$

$32\tan^2\alpha + 182\tan\alpha - 88 = 0$

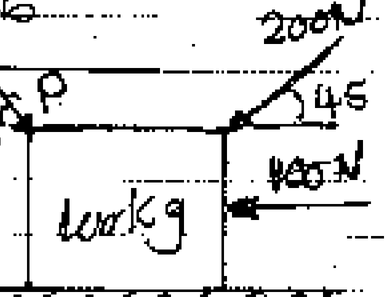
$\alpha = 80.74$ or $\alpha = -24.13$

when $\alpha = 80.74 \Rightarrow R = 139.57$ lb

when $\alpha = -24.13 \Rightarrow R = 10.74$ lb

90 lb
70 sinα
70 cosα
130 sinα
130 cosα

ex.6 An (100kg) block rests on a horizontal plane and subjected to a forces as shown. Find the magnitude of the force (P) required to give the block an acceleration of (3 m/s2) to the right. The coefficient of friction (kinetic friction) between the block and the plane is ($\mu_K = 0.25$). Take ($g = 10$ m/s2).



solu

$W = mg = (100kg)(10 m/s^2) \Rightarrow W = 1000N$

Friction force $(F_K) = \mu_K n \Rightarrow F_K = 0.25n$

$+\uparrow \Sigma F_y = n - 1000 - P\sin 30 - 200\sin 45 = 0$

$n = 1141.42 + 0.5P$ ——(1)

$\rightarrow \Sigma F_x = ma$, $P\cos 30 - 200\cos 45 - 100 - 0.25n = 100 * 3$

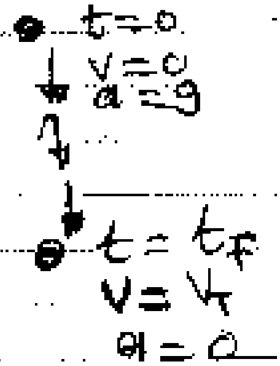
$0.866P - 141.42 - 100 - 0.25n = 300$ —— (2)

substitute equ (1) in equ (2)

$P = 1115.75\,N$ or $P = 1.15\,KN$

Ex.7

A small piece of styrofoam packing material is dropped from a height of (2m) above the ground. Until it reaches terminal speed, the magnitude of its acceleration is given by ($a = g - bV$). After falling (0.5m), the styrofoam effectively reaches terminal speed and then takes (5 sec) to reach the ground. (a) What is the value of the constant (b). (b) What is the acceleration at ($t=0$)? (c) What is the acceleration when the speed is (0.150 m/s)? Take: $g = 9.80\ m/s^2$



Solu (b) At $t=0$, $a = g = 9.80\ m/s^2$

(a) $a = g - bV$

at $t=0$, $V=0$, $a=g$

at $V = V_t$, $a=0$

$0 = g - bV_t \Rightarrow b = \frac{g}{V_t}$

at $y_f = 0.5m$, $V = V_t$, time $= t$, $a=0$

$y_f = y_i + v_{yi}t + \frac{1}{2}a_y t^2$

$0.5 = 0 + V_t t + 0 \Rightarrow t = \frac{0.5}{V_t}$ —— (1)

at $y_f = 2m$, $V = V_T$, time $= (t+5)$, $a=0$

$2 = 0 + V_T(t+5) + 0 \Rightarrow 2 = V_t t + 5V_T$ —— (2)

subs equ (1) in equ (2)

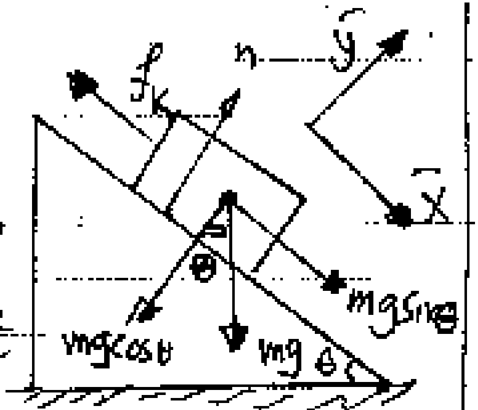
$V_T = 0.3\ m/s$, $b = g/V_t = (9.80)/0.3 \Rightarrow b = 32.75$

(c)

$a = g - bV = 9.80 - (32.75)(0.15) \Rightarrow a = 4.90\ m/s^2$

(ex,8)

A (3kg) block starts from rest at the top of (30°) incline and slides at a distance of (2m), down the incline in (1.50) second. Find ⓐ the magnitude of block acceleration ⓑ the coefficient of friction between block and plane, ⓒ the friction force acting on the block and ⓓ the speed of the block after it has slid (2 m). Take: $g = 9.80\ m/s^2$



(Solu)

ⓐ $X_f = x_i + U_{xi}t + \frac{1}{2}a_x t^2$, $X_i = 0$, $X_f = 2m$, $V_{xi} = 0$, $t = 1.5\ s$

$2 = 0 + 0 + \frac{1}{2}a_x(1.5)^2 \Rightarrow a_x = 1.78\ m/s^2$

ⓒ

$\Sigma F_x = mg\sin\theta - f_k = m a_x$

$(3kg)(9.80)(\sin 30) - f_k = (3kg)(1.78\ m/s^2)$

$f_k = 9.36N$

ⓑ

$f_k = \mu_k n$

$\Sigma F_y = 0$, $n - mg\cos\theta = 0 \Rightarrow n = (3)(9.80)(\cos 30) = 25.46N$

$\mu_k = f_k/n = 9.36N/25.46N \Rightarrow \mu_k = 0.368$

ⓓ $V_{xf} = ?$, $V_{xi} = 0$, $t = 1.5\ sec$

$V_{xf} = U_{xi} + a_x t_f \Rightarrow V_{xf} = 0 + (1.78)(1.5) \Rightarrow V_{xf} = 2.67\ m/s$

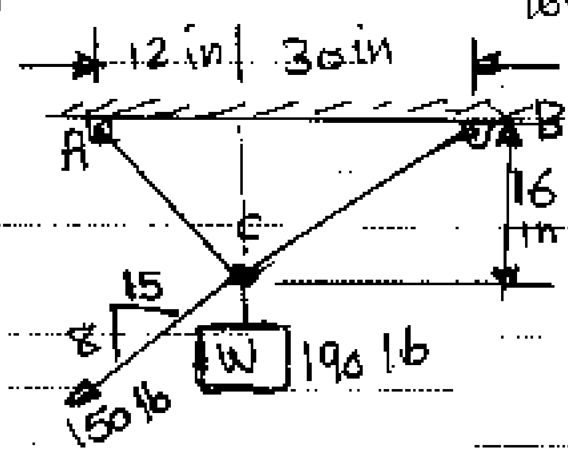
(or)

$V_f^2 = U_i^2 + 2a_x(X_f - X_i) \Rightarrow V_{xf}^2 = 0 + (1.78)(2-0)$

$V_f^2 = 7.12 \Rightarrow V_f = 2.67\ m/s$

(ex.9)

Two cables tied together at ⓒ, are loaded as shown. Knowing that (W = 190 lb), determine the tension (a) in cable (AC), (b) in cable BC.

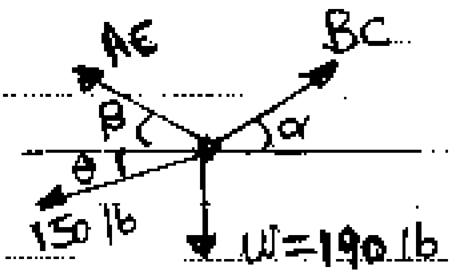


Solu

$\tan\alpha = \frac{16}{30} \Rightarrow \alpha = 28.07^\circ$

$\tan\beta = \frac{16}{12} \Rightarrow \beta = 53.13^\circ$

$\tan\theta = \frac{8}{15} \Rightarrow \theta = 28.07^\circ$



$\sum F_x = 0$

$CB\cos\alpha - CA\cos\beta - 150\cos\theta = 0$

$CA = -220.58 + 1.47\,CB$ ---- (1)

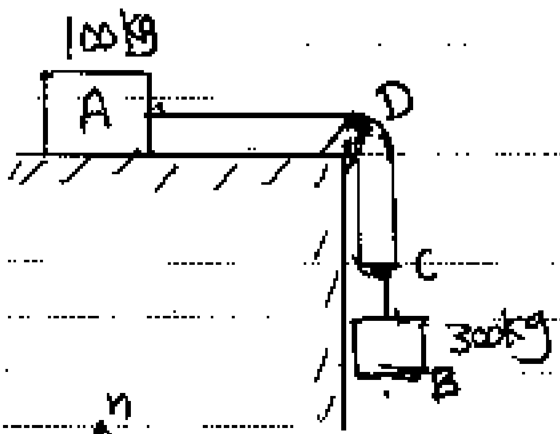
$\sum F_y = 0$

$CB\sin\alpha + CA\sin\beta = 190 + 150\sin\theta$ ---- (2)

Subs equ (1) in equ (2)

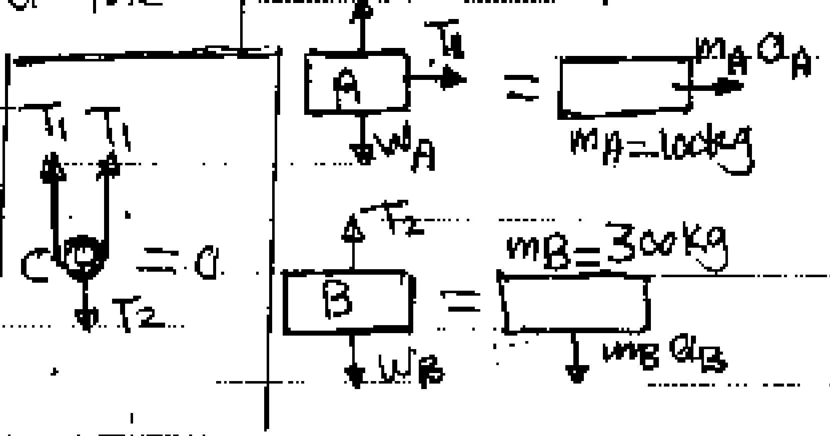
$CB = 265.51$ lb and $CA = 169.73$ lb

Ex. 10 Two blocks shown start from rest. The horizontal plane and the pulley are frictionless, and the pulley is assumed to be of negligible mass. Determine the acceleration of each block and the tension in each cord. Use: $g = 9.81\ m/s^2$



Solu

If the block (A) moves through x_A to the right block (B) moves



down through $X_B = 2X_A$

Differentiating twice with respect to (t) we have : $a_B = \frac{1}{2} a_A$ ---(1)

Block (A)

$\xrightarrow{+} \Sigma F_x = m_A a_A \Rightarrow T_1 = m_A a_A$ ---(2)

Block (B)

$W_B = m_B g = (300kg)(9.81 m/s^2) \Rightarrow W_B = 2940 N$

$+\downarrow \Sigma F_y \quad 2940 - T_2 = 300 a_B$

or $2940 - T_2 = 300 (\frac{1}{2} a_A)$

$T_2 = 2940 - 150 a_A$ ---(3)

Pulley (C)

Since m_c is assumed to be zero, we have:

$+\downarrow \Sigma F_y = m_c a_c = 0 \Rightarrow T_2 = 2T_1$ ---(4)

subs for T_1 and T_2 from equs (2), (3) respectively into equ (4) we have :-

$2940 - 150 a_A - 2(100 a_A) = 0 \Rightarrow a_A = 8.4 m/s^2$

subs this value into equs (1) and (2) we have

$a_B = 4.20 m/s^2$ and $T_1 = 840 N$

From equ (4), $T_2 = 1680 N$

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ex. 4

A person delivering news papers covers his rout by traveling (100m) South, (150m) east, and then (200m) north. (a) what is his resultant displacement and what is the total distance he travels? (b) find the average speed, average velocity for this (12 minute) trip.

Solu)

$\vec{A_x} = 0, \ \vec{A_y} = -100m$

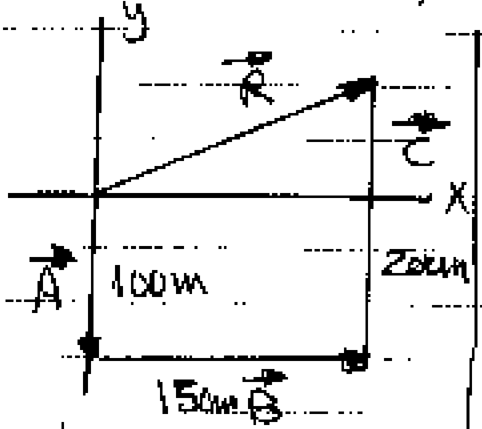
$\vec{B}_x = +150m$, $\vec{B}_y = 0$

$\vec{C}_x = 0$, $\vec{C}_y = +200m$

$\vec{R}_x = \vec{A}_x + \vec{B}_x + \vec{C}_x = 0 + 150 + 0 = 150m$

$\vec{R}_y = \vec{A}_y + \vec{B}_y + \vec{C}_y = -100 + 0 + 200m = 100m$

$\vec{R} = \vec{R}_x + \vec{R}_y = 150\hat{i} + 100\hat{j}$



ⓐ $R = \sqrt{(R_x)^2 + (R_y)^2} = \sqrt{(150)^2 + (100)^2}$

$= 180.27$ m displacement

Total distance $= 100 + 150 + 200 = 450m$

$\tan\theta = R_y / R_x \Rightarrow \theta = \tan^{-1} 100/150 \Rightarrow \theta = 33.69°$ north of east

ⓑ Average speed $= \dfrac{\text{distance}}{\text{time}} = \dfrac{450}{12 \times 60} = 0.625\ m/s$

Average velocity $= \dfrac{\text{displacement}}{\text{time}} = \dfrac{180.27}{12 \times 60} = 0.25\ m/s$

ex.12

The force exerted by the wind on the sails of a sailboat is (390N) north. The water exerts a force of (180N) east. If the boat has a mass of (270 kg), what are the magnitude and direction of its acceleration ?

solu

Since the two forces are perpendicular

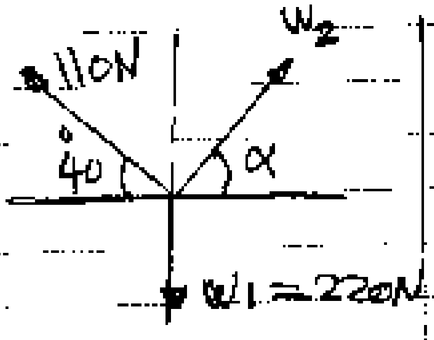
$F_R = \sqrt{(180)^2 + (390)^2} = 430N$ at, $\tan\theta = \dfrac{390}{180} \Rightarrow \theta =$

$65.2°$ north of east.

$a = F_R / m = \dfrac{430}{270} = 1.59\ m/s^2$

17/5 1/2

ex.13 Find the value of w_2 and α.



Solu

$\Sigma F_x = 0 = w_2 \cos\alpha - 110 \cos 40$

$w_2 \cos\alpha = 84.2$ ---- (1)

$\Sigma F_y = 0$

$220 = 110 \sin 40 + w_2 \sin\alpha$

$w_2 \sin\alpha = 149.3$ ---- (2)

Divided equ② / equ①

$\tan\alpha = 1.773 \Rightarrow \alpha = 60.57°$

$w_2 = 84.2 / \cos 60.57 \Rightarrow w_2 = 171.36N$

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ex.14

A bag of sugar weights (5 lb) on Earth, what would it weight in newtons on the Moon where ($g = 1/6$) th an on Earth. Repeat for Jupiter where ($g = 2.64$) times that on Earth. Find the mass of the bag of sugar in kilograms at each of the three locations.

Take $g_E = 9.80 m/s^2$, $1b = 4.448N$

Solu

The weight of the bag of sugar on Earth is:-

$w_E = m\, g_E = (5\ lb)(4.448\ N/lb) = 22.2N$

$w_M / w_E = m g_M / m g_E = g_M / g_E \Rightarrow w_M = w_E (g_M / g_E)$

$w_M = (22.2N)(1/6) = 3.70N$

On Jupiter $w_J = w_E (g_J / g_E) = (22.2N)(2.64) = 58.6N$

The mass is the same at all three locations.

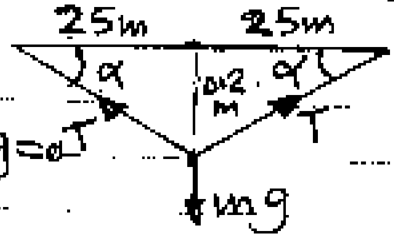
$$m = w_E / g_E = \frac{(5\ lb)(4.448\ N/lb)}{9.80\ m/s^2} = 2.27\ Kg$$

ex.15

A distance between two telephone poles is (50 m). When a (1 kg) bird lands on the telephone wire midway between the poles, the wire sag (0.20 m). How much tension does the bird produce in the wire? (Ignore the weight of the wire). take $g = 9.80\ m/s^2$

solu

$\alpha = \tan^{-1} \frac{0.2}{25} \Rightarrow \alpha = 0.458^{\circ}$



$a_y = 0$, $\Sigma F_y = T\sin\alpha + T\sin\alpha - mg = 0$

giving $2T\sin\alpha = mg$ or

$T = \frac{9.80}{2\sin 0.458} \Rightarrow T = 613\ N$

ex.16

A boat moves through the water with two forces acting on it. one is (2000 N) forward push by the water and the other is a (1800 N) resistive force due to the water around the bow. ⓐ what is the acceleration of the (1000 kg) boat ⓑ if it starts from rest, how far will the boat move in (10 s)? ⓒ what will its velocity be at the end of that time?

solu

ⓐ $a = \frac{\Sigma F}{m} = \frac{(2000 - 1800)}{1000\ kg} = 0.2\ m/s^2$

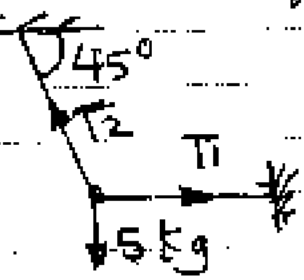
ⓑ $\Delta x = v_0 t + \frac{1}{2} a t^2 = 0 + \frac{1}{2}(0.2\ m/s^2)(10\ s)^2 = 10\ m$

ⓒ $v_f = v_0 + at = 0 + (0.2\ m/s^2)(10\ s) \Rightarrow v_f = 2\ m/s$

ex.17

A weight of (5 kg) is suspended as shown.

calculate the value of tensions (T_1, T_2). Take $g = 10\, m/s^2$



Solu

$\Sigma F_x = 0, \quad T_1 = T_2 \cos 45$

$T_1 = 0.707\, T_2$

$W = (5\, kg)(10\, m/s^2) = 50\, N$

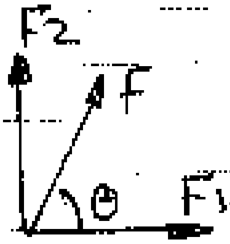
$\Sigma F_y = 0, \quad 50 = T_2 \sin 45$

$T_2 = 70.71\, N, \quad T_1 = 0.707(70.71) \Rightarrow T_1 = 50\, N$

ex. 18

A body of mass (5 kg) is acted upon by two perpendicular forces of (8N) and (6N). Give the magnitude and direction of the acceleration of the body.

Solu



$F_1 = 8N, \quad F_2 = 6N, \quad m = 5\, kg$

$F = \sqrt{F_1^2 + F_2^2} = \sqrt{8^2 + 6^2} \Rightarrow F = 10\, N$

$\Sigma F = ma$

$a = F/m = 10/5 = 2\, m/s^2$

$\tan\theta = F_2/F_1 \Rightarrow \theta = \tan^{-1} 6/8 \Rightarrow \theta = 36.86$

ex. 19

Three blocks are connected by strings as shown and are pulled by a force of ($T_3 = 150N$). If $m_1 = 5\, kg$, $m_2 = 10\, kg$ and $m_3 = 15\, kg$, calculate the acceleration of the system and the value of T_1, T_2.

Solu

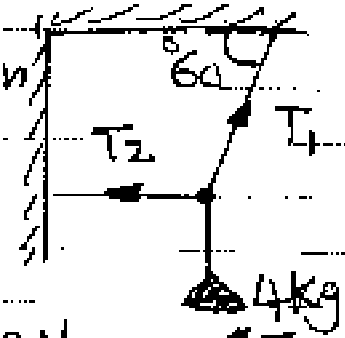
Acceleration of the system, $a = \dfrac{F}{m_1 + m_2 + m_3} = \dfrac{150}{5+10+15} = 5\, m/s^2$

$T_1 = m_1 a = 5 \times 5 = 25\, N$

$T_2 = (m_1 + m_2) a = (5+10)5 = 75\, N$

ex.20

Determine tension T_1 and T_2 shown in the figure. Take: $g = 10\,m/s^2$



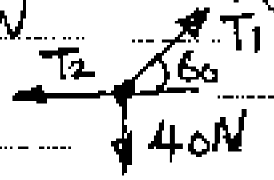
solu

$\Sigma Fx = 0$, $T_1 \cos 60 - T_2 = 0$

$T_1 = 2T_2$ ——— (1)

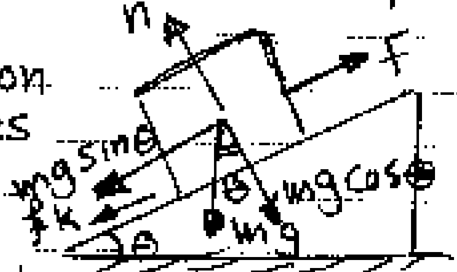
$\Sigma Fy = 0$, $T_1 \sin 60 - 40 = 0 \Rightarrow T_1 = 46.2N$

$T_2 = 23.1\,N$



ex.21

A block of mass (2 Kg) rests on an inclined plane which makes an angle of (30°) with the horizontal. What force should be applied to the back so that it moves down without any acceleration, if $\mu = \sqrt{3}/2$ and $g = 9.8\,m/s^2$



solu

$\Sigma Fx = 0$, $F - mg \sin\theta - f_k = 0$, $f_k = \mu n = \mu mg \cos\theta$

$F = mg \sin\theta + \mu mg \cos\theta = mg(\sin\theta + \mu \cos\theta)$

$F = (2)(9.80)(\sin 30 + \sqrt{\tfrac{3}{2}} \cos 30)$

$F =$ ..... N

ex.22

Find the distance traveled by a body before coming to rest if it is moving on the ground with a velocity of (36 Km/h) and the coefficient of friction between the body and the ground is (0.2). Take: $g = 9.80\,m/s^2$

solu

$$V = 36\,Km/h = \frac{36 * 1000}{3600} = 10\,m/s$$

$f_k = \mu_k n = F = ma$, $n = mg$

$\cancel{m}a = \mu \cancel{m} g \Rightarrow a = \mu g = (0.2)(9.8) = 1.96 m/s^2$

$V_{xf}^2 = U_{xi}^2 + 2a_x(x_f - x_i)$

$0 = 10^2 + 2(-1.96)(x_f - 0)$

$x_f = 25.51 m$

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ex.23

A motor car running at the rate of (7 m/s) can be stopped by its brakes in (10m). Prove that the resistance to the car's motion, when the brakes are on is (1/4) of the weight of the care.

Take: $g = 9.80 m/s^2$

Solu

$v_i = 7 m/s$, $v_f = 0$, $x_f = 10m$, $x_i = 0$

$v_{xf}^2 = u_{xi}^2 + 2a_x(x_f - x_i) \Rightarrow 0 = 7^2 + 2a_x(10 - 0)$

$a_x = -2.45 m/s^2$

$F = ma \Rightarrow F = (2.45 m) N$ and $w = mg = (9.8m) N$

$m = F/2.45 = w/9.80$

$\therefore \frac{F}{2.45} = \frac{w}{9.80} \Rightarrow F = w/4$

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ex.24

If an object of mass (m) moves with constant velocity (v), the net force on the object is (a) mg (b) mv (c) ma (d) zero (e) none of these answers is correct.

Solu

An object in equilibrium has zero acceleration ($\vec{a} = 0$) so that the magnitude and direction of the object's velocity must be constant. Also Newton's second law states that the net force acting on an object in equilibrium is zero. The only untrue statement among the given choices is (d) untrue because the value of the velocity's constant magnitude need not to be zero.